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Problem Sheet 1

Differential Geometry I

WS 2018/2019

Discussion in the Exercise class 22.10./29.10.

Problem 1

Let X be an uncountable set and τ_{abz} the set family

$$\tau_{abz} := \{\emptyset\} \cup \{X \setminus A \mid A \text{ countable}^*\}.$$

Prove that (X, τ_{abz}) is a topological space and determine the interior $int(M)$, the closure $cl(M)$ and the boundary ∂M for a set $M \subset X$.

* *In this lecture by a countable set we mean a set which is empty, finite or proper countable.*

Problem 2

Show:

a) Every second countable topological space is separable.

b) Every separable metric space is second countable.

Problem 3

On the set $\mathbb{R}$ of real numbers (with standard topology) we consider the following equivalence relation:

$$x \sim y \iff x = y \text{ or } x, y \in \mathbb{Z}.$$

Show that the point $[0]$ in the factor space $\mathbb{R}/_\sim$ does not admit a countable basis of neighbourhoods.

Problem 4 *The Sorgenfrey line*

We consider the set of real numbers $\mathbb{R}$ and the following family τ_{sorg} of subsets of $\mathbb{R}$:

$$U \in \tau_{sorg} \quad :\iff \quad U = \emptyset \text{ or } U \text{ is the union of arbitrary many intervalls of the form } [a, b),\ -\infty < a < b < +\infty.$$

Show:

a) $(\mathbb{R}, \tau_{sorg})$ is a topological space.

b) $(\mathbb{R}, \tau_{sorg})$ is separable.

c) $(\mathbb{R}, \tau_{sorg})$ is first countable.

d) $(\mathbb{R}, \tau_{sorg})$ is not second countable.

e) $(\mathbb{R}, \tau_{sorg})$ is not metrizable.