Homework 10

Topology II

Winter 2016/17

See also Problem 2 and 3 from Homework 9

Problem 1
Compute the cohomology rings $H^*(\mathbb{R}P^n;\mathbb{Z})$ and $H^*(\mathbb{R}P^\infty;\mathbb{Z})$.

Problem 2
(1) Show that $H^*(\coprod_\alpha X_\alpha; R) \cong \oplus_\alpha H^*(X_\alpha; R)$ as rings.
(2) Show that $\tilde{H}^*(\bigvee_\alpha X_\alpha; R) \cong \oplus_\alpha \tilde{H}^*(X_\alpha; R)$ as rings if the base points $x_\alpha \in X_\alpha$ used for the construction of the wedge admit an open neighborhood which contracts onto x_α.
(3) Show that $\mathbb{R}P^3$ is not homotopy equivalent to $\mathbb{R}P^2 \vee S^3$. Compute their homology and cohomology groups with various coefficients.