NUMERICAL TREATMENT OF A STOCHASTIC PROGRAMMING MODEL FOR OPTIMAL POWER DISPATCH

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ABSTRACT. The economic dispatch of electric power with uncertain demand is modelled as a stochastic program with simple recourse. The unknown distribution functions of the demand are approximated by smooth nonparametric estimates. We discuss the numerical treatment of the model and report on computational results.

1. The stochastic power dispatch model

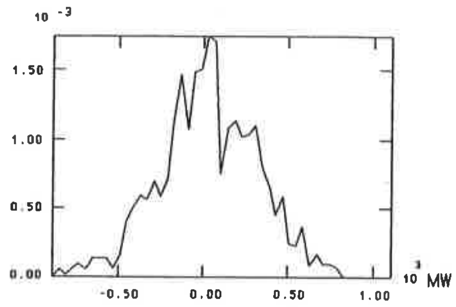
We consider the optimal scheduling of power generation to the units of an energy production system, aiming at minimizing the total generation costs while the power demand is met and certain operational constraints of the system are satisfied. This process is often divided into a three-stage procedure: (1) unit commitment for the base-load-plants, (2) power dispatch for the operating cycle and (3) short-term load dispatch for the base-load-plants (optional). The peculiarities of the energy production system and of the power dispatch model we shall consider are the following:

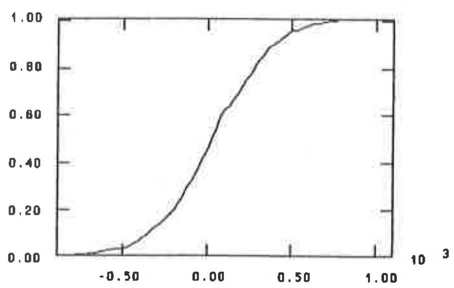
(a) The system consists of thermal power stations *(tps)*, pumped hydro storage plants *(psp)* and an energy contract with connected systems,
(b) *tps* and *psp* serve as base-and peak-load plants, respectively,
(c) the model is designed for a daily operating cycle and assumes that a unit commitment stage has been carried out before,
(d) the cost functions of the *tps* are taken to be strictly convex and quadratic,
(e) the transmission losses are modelled by means of adjusted portions of the demand,
(f) the model takes explicit account of the uncertainty of the electric power demand.

The mathematical formulation of the optimal power dispatch model reads

$$\min\{g(x) : x \in C, Ax = z\}. \tag{1.1}$$

Here, the components of x are the outputs (inputs) of the *tps* and *psp* at each interval of a discretization of the time-horizon and the levels of electric power which correspond to the energy contract at each time interval. The total generation cost function g is convex quadratic, the portion which corresponds to the contract is a linear function. The set C is a nonempty bounded convex polyhedron formed by the operational constraints of the system, e.g. bounds for the power output of the plants, balances between generation and pumping





Figures 1 and 2. Estimates for the density and distribution function

in the *psp*, balances for the *psp* over the whole time-horizon, fuel quotas for the *tps* etc. At this instant the equation $Ax = z$ in (1.1) means that the total generated output $[Ax]_r$ meets the electricity demand $z_r, r = 1, \ldots, N$, at each time interval. A detailed description of the model can be found in [2] and [6].

Now, we consider the demand z as a random vector and denote by F_r the probability distribution function of $z_r, r = 1, \ldots, N$. As in [1], [2] we introduce penalty costs for the deviation of the scheduled output from the actual demand for under- and overdispatching, respectively. Adding the expected penalty (or recourse) costs to the deterministic objective function g we obtain the following stochastic power dispatch model:

$$\min\{g(x) + \sum_{r=1}^{N} \int_{-\infty}^{+\infty} Q_r(t - \chi_r) dF_r(t) : x \in C, Ax = \chi\} , \tag{1.2}$$

where

$$Q_r(t) := \begin{cases} q_r^+ t, & t \geq 0 \\ -q_r^- t, & t < 0 \end{cases}$$

and q_r^+, q_r^- denoting the recourse costs for under- and overdispatching at the r-th time interval, respectively. For a discussion of the interpretation and choice of the recourse costs we refer to [1]. More information on various aspects of power dispatch can be found in [8]. It is well-known that (1.2) is a particular stochastic program with simple recourse (cf. [3]). Under weak assumptions, (1.2) is a (large scale) convex nonlinear program having linear constraints and C^1- data (if all distribution functions F_r have densities). For our application of the model to the electricity sector of East Germany the dimension of the vector x is 840. For the uncertain demand, a set of empirical data is given. In [1] it is suggested that the distribution functions F_r can be chosen as (trimmed) normal. However, our tests with the available empirical data have not justified this hypothesis. The Figs. 1 and 2 show estimates for the density and distribution function of the centered demand during 1 p.m.-2 p.m. of a day of normal category. The estimates are obtained by a nonparametric kernel estimator according to (2.1).

2. Numerical treatment, results and conclusions

For the numerical solution of the stochastic power dispatch model (1.2) we first replace the distribution functions F_r by the following smooth nonparametric estimates $\hat{F}_r$ ($r = 1, \ldots, N$). Let $z_{1r}, \ldots, z_{nr}, \ldots$ be an independent sample from the distribution F_r ($r = 1, \ldots, N$), $k : \mathrm{R} \to \mathrm{R}$ a nonnegative function having the property $\int_{\mathrm{R}} k(t)dt = 1$ ("*kernel*"), and (b_n) a sequence of positive numbers tending to zero (" *smoothing parameters*"). Then we consider *kernel estimates* of F_r ($r = 1, \ldots, N$)

$$\hat{F}_r(t) := \frac{1}{nb_n} \sum_{i=1}^{n} \int_{-\infty}^{t} k\left(\frac{x - z_{ir}}{b_n}\right) dx \quad (t \in \mathrm{R}\,;\, n \in \mathrm{N}). \tag{2.1}$$

For more information and background on kernel-type estimators, especially on possible choices of k and (b_n), and on asymptotic results for $n \to \infty$, we refer to [7],[9]. These asymptotic arguments together with stability results for stochastic programs yield a theoretical foundation of our approach (see [2]). The use of the estimates $\hat{F}_r$ for the distribution functions of the electricity demand leads to the following convex program having a continuously differentiable objective and linear constraints:

$$\min\{g(x) + \hat{Q}(\chi) : x \in C, Ax = \chi\} \tag{2.2}$$

Explicit formulas for $\hat{Q}$ and its gradient can be derived easily (see Sect. 5 in [2]). They show the dependence of $\hat{Q}$ on the recourse costs, the samples and smoothing parameters, and on the real functions $\mathcal{K}_1(t) := \int_{-\infty}^{t} k(u)du$, $\mathcal{K}_2(t) := \int_{-\infty}^{t} uk(u)du$. Since, for most kernels k, $\mathcal{K}_1$ and $\mathcal{K}_2$ can be calculated explicitly, no numerical integration has to be performed when evaluating $\hat{Q}$ and its gradient.

The second step in our treatment of (1.2) is the solution of (2.2) by standard NLP-techniques. A program system `STOCHOPT` following this approach has been developed by using the NLP-code `MINOS` (see [5]). The system is written in FORTRAN 77 (algorithm) and TURBO-PASCAL (user-interface). The first version of `STOCHOPT` was implemented on an IBM PC 386 and first tested for a stochastic aircraft allocation problem (see [2]). Recently, a series of test runs for the stochastic power dispatch model (of East Germany) were performed. During all experiments the running time on a PC 386 did not exceed 5 minutes and had the same order of magnitude as for the corresponding deterministic model. The reason for this surprising effect is the low number of iterations of the nonlinear programming algorithm, which is (probably) due to the strong convexity of the recourse cost function $\hat{Q}$. Another surprising result is illustrated in Fig.3, showing the quotient QUO of the optimal costs of the stochastic model over that for the deterministic one as a function of the prescribed reserve level RL (%) for the demand, and elucidating the fact that the optimal solution of the stochastic model is superior to that of the (corresponding) deterministic one even if a reserve level of only 3% of the demand is adjusted.

QUO	*RL*
1.05	0%
0.97	3%
0.94	5%

Figure 3.

A more detailed comparison of (stochastic and deterministic) output schedules of all generation units shows that, for the "stochastic solution", high-cost (low-cost) units operate (as long as possible) at the lower (upper) output level. Fig.4 shows the daily output schedule for a high-cost unit (Boxberg2). An additional observation is that the total number of

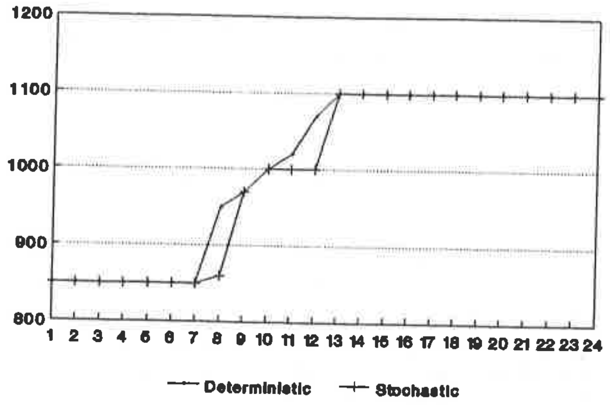


Figure 4. Comparison of a stochastic and deterministic output schedule

regulations of all units (during the whole time period) is smaller in case of the "stochastic solution".

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