Stochastic Programming for Power Production and Trading Under Uncertainty

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Abstract. Optimization models under uncertainty for mid-term cost-optimal operation of a hydro-thermal power system and for simultaneous power production and day-ahead trading at a power exchange are presented. Algorithms for solving these models are sketched and initial numerical experience is reported.

1 Introduction

Uncertainty is of increased importance in today's power management. Traditional sources of uncertainty, such as weather conditions infecting the load curve, are accompanied by novel sources that are due to the liberalization of power markets. Trading now becomes an issue, and new market instruments arise that, to more or less extent, involve some level of uncertainty.

In the present paper we report on two mathematical optimization models that are tailored to coping with randomness in power production and power trading. The paper grew out of a cooperation with the VEAG Vereinigte Energiewerke AG Berlin, a utility running a hydro-thermal power system in the Eastern part of Germany. Our mathematical instruments stem from (mainly two-stage) stochastic integer programming.

In Sect. 2 we present basic issues of mathematical modelling of generation units and their interaction in the VEAG system. Section 3 then addresses a planning model for mid-term cost optimal power production under uncertainty of electrical load, fuel prices and electricity prices in power contracts. In Sect. 4 we develop some ideas on how to couple day-ahead trading at a power exchange with the basic production model from Sect. 2. In both Sects. 3 and 4 we place accent on modelling issues, sketch the optimization algorithms for problem solution, and display initial numerical experience.

2 Power Production – The VEAG System

The generation system of VEAG Vereinigte Energiewerke AG Berlin comprises pumped-storage hydro as well as coal and gas fired thermal power

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plants. Throughout, we work with a (mostly hourly) discretization of the planning horizon into $t = 1, ..., T$ subintervals, and we assume that there are I thermal as well as J pumped-storage hydro units. The variable $u_{it} \in \{0,1\}$, $i = 1, \dots, I; t = 1, \dots, T$ indicates whether the thermal unit i is in operation at time t. Variables p_{it}, s_{jt}, w_{jt}, $i = 1, \dots, I$, $j = 1, \dots, J$; $t = 1, \dots, T$ are the output levels for the thermal units, the hydro units in generation and in pumping modes, respectively. The variables l_{jt} denote the fill (in energy) of the upper dam of the hydro unit j at the end of interval t.

The power output of units and the fill of the upper dams have to fit the following bounds

$$
\begin{array}{rl}
p_{it}^{min} u_{it} \leq p_{it} \leq p_{it}^{max} u_{it}, & i = 1, \dots, I, \ t = 1, \dots, T \\
0 \leq s_{jt} \leq s_{jt}^{max}, & j = 1, \dots, J, \ t = 1, \dots, T \\
0 \leq w_{jt} \leq w_{jt}^{max}, & j = 1, \dots, J, \ t = 1, \dots, T \\
0 \leq l_{jt} \leq l_{jt}^{max}, & j = 1, \dots, J, \ t = 1, \dots, T
\end{array}
\tag{1}
$$

Here, $p_{it}^{min}, p_{it}^{max}, s_{jt}^{max}, w_{jt}^{max}$ denote minimal and maximal outputs, respectively, and l_j^{max} is the maximal fill of the upper dam. Load coverage is modeled by the constraints

$$
\sum_{i=1}^{I} p_{it} + \sum_{j=1}^{J} (s_{jt} - w_{jt}) \geq d_t, \quad t = 1, \dots, T, \tag{2}
$$

where d_t denotes the electrical load at time t.

In addition to load coverage, reserve management is a key issue in power production. Quite different reserve schemes are employed in practice. At least the following requirement involving a so-called spinning reserve r_t has to be met for the thermal units:

$$
\sum_{i=1}^{I} (u_{it} p_{it}^{max} - p_{it}) \geq r_t, \quad t = 1, \dots, T. \tag{3}
$$

For the pumped-storage plants we have the following balances that interconnect different time intervals:

$$
\left.
\begin{array}{ll}
l_{jt} = l_{j,t-1} - (s_{jt} - \eta_j w_{jt}), & \\
l_{j0} = l_j^{in}, & l_{jT} = l_j^{end},
\end{array}
\right\} \quad j = 1, \dots, J, \ t = 1, \dots, T. \tag{4}
$$

Here, l_j^{in}, l_j^{end} are the initial and final fills (in energy) of the upper dams, η_j denote the pumping efficiencies. Constraints avoiding simultaneous generation and pumping in the hydro plants are dispensable since it can be shown that such a deficiency can not occur in optimal points.

To avoid excessive thermal stresses in the coal fired blocks, they have to adhere to minimum up and down times σ_i and τ_i. These are modeled via

$$
\begin{array}{ll}
u_{it} - u_{i,t-1} \leq u_{i\sigma}, & \sigma = t+1, \dots, \min\{t + \sigma_i - 1, T\}, \\
u_{i,t-1} - u_{it} \leq 1 - u_{i\tau}, & \tau = t+1, \dots, \min\{t + \tau_i - 1, T\}, \\
 & i = 1, \dots, I; t = 2, \dots, T-1.
\end{array}
\tag{5}
$$

The constraints (1)–(5) provide a mathematical model for basic features and basic interaction of the generating units of the VEAG system. Typical objective functions to be minimized on the constraint set (1)–(5) concern fuel consumption for starting up and operating thermal units together with costs (or revenues) according to power contracts. Formalizing fuel cost minimization, for instance, leads to the objective function

$$\sum_{t=1}^{T}\sum_{i=1}^{I} C_{it}(p_{it}, u_{it}) + \sum_{t=1}^{T}\sum_{i=1}^{I} S_{it}(u_i), \tag{6}$$

where C_{it} denote the piecewise linear fuel costs and $S_{it}(u_i) = c_{it}\max\{u_{it} - u_{i,t-1}, 0\}$ the start-up costs for the thermal unit i, where $u_i := (u_{it})_{t=0}^{T}$ and u_{i0} is a given initial value.

In Sects. 3 and 4, the above mathematical apparatus will be utilized as starting point for including the sources of uncertainty outlined in the introduction.

3 Mid-Term Power Planning

For the system introduced in Sect. 2 we describe a model for its mid-term cost-optimal power production planning under uncertainty on the electrical load and on the electricity and fuel prices. For mid-term planning models we are faced with stochastic data when considering time periods lying far in the future. In order to derive solutions that hedge against uncertainty it is necessary to incorporate the randomness of the data into the model. So far this is mainly done for operational models (cf. [4,6] and the references therein).

Since we regard future planning periods (e. g. next week or year), we assume that the quality of available information on the load uncertainty does not depend on time, i.e., it does not increase with the length of the planning horizon. Furthermore, the load is stochastic right from the beginning of the considered time period. The stochastic behaviour of the load $\boldsymbol{d}_t$, the spinning reserve $\boldsymbol{r}_t$ and the price for fuel and electricity – characterized by its coefficients $\boldsymbol{a}_t$, $\boldsymbol{b}_t$ and $\boldsymbol{c}_t$ – is represented by a discrete-time stochastic process $\{\boldsymbol{\xi}_t := (\boldsymbol{d}_t, \boldsymbol{r}_t, \boldsymbol{a}_t, \boldsymbol{b}_t, \boldsymbol{c}_t)\}_{t=1}^{T}$on some probability space $(\Omega, \mathcal{A}, \mathbb{P})$. Now, the decision process consists of two stages where the first-stage decisions correspond to the here-and-now schedules for all power generation units. The second-stage decisions, on the other hand, correspond to future compensation or recourse actions of each unit in each time period. The latter naturally depend on the environment created by the first-stage decisions and the load and price scenarios in that specific time period. Hence, the aim of such a two-stage planning model can be formulated as follows. Find an optimal schedule for the whole system and planning horizon such that the uncertain data can be compensated by the system, all system constraints are satisfied and the sum of the total generation costs and the expected recourse costs is minimal.

In order to give a mathematical formulation of the two-stage model let $(u,\ p,\ s,\ w)$ denote the first-stage decisions and $(\mathbf{u},\ \mathbf{p},\ \mathbf{s},\ \mathbf{w})$ be the stochastic second-stage decisions having the components $\mathbf{u}_{it},\ \mathbf{p}_{it},\ \mathbf{s}_{jt},\ \mathbf{w}_{jt}$, which correspond to the recourse actions of each unit at time period t. In addition to the (non-stochastic) constraints for $(u,\ p,\ s,\ w)$, i.e. the capacity limits (1), the storage dynamics (4) and the minimum up- and down-times (5), we have to require that the recourse actions also satisfy the system constraints. These are the operating ranges of all units, the minimum up/down-time requirements for the thermal units and the reservoir capacity bounds:

$$p_i^{min}\mathbf{u}_{it} \le \mathbf{p}_{it} \le p_i^{max}\mathbf{u}_{it},\ \mathbf{u}_{it} \in \{0,1\}, \tag{7a}$$

$$\begin{gathered} \mathbf{u}_{it} - \mathbf{u}_{i,t-1} \le \mathbf{u}_{i,\sigma},\ \sigma = t+1,\dots,\min\{t+\sigma_i-1,\ T\}, \\ \mathbf{u}_{i,t-1} - \mathbf{u}_{it} \le 1 - \mathbf{u}_{i,\tau},\ \tau = t+1,\dots,\min\{t+\tau_i-1,\ T\}, \\ t = 2,\dots,T-1,\ i = 1,\dots,I,\ I\!P\text{ - a. s.} \end{gathered} \tag{7b}$$

$$\begin{gathered} 0 \le \mathbf{s}_{jt} \le s_j^{max},\ 0 \le \mathbf{w}_{jt} \le w_j^{max},\ 0 \le \boldsymbol{\ell}_{jt} \le \ell_{jt}^{max}, \\ t = 1,\dots,T,\ j = 1,\dots,J,\ I\!P\text{ - a. s.} \end{gathered} \tag{7c}$$

$$\begin{gathered} \boldsymbol{\ell}_{jt} = \boldsymbol{\ell}_{j,t-1} - \mathbf{s}_{jt} + \eta_j \mathbf{w}_{jt},\ t = 1,\dots,T, \\ \boldsymbol{\ell}_{j0} = \ell_j^{in},\ \boldsymbol{\ell}_{jT} = \ell_j^{end},\ j = 1,\dots,J,\ I\!P\text{ - a. s.} \end{gathered} \tag{7d}$$

Here some remarks concerning the interplay of the two stages are due. The first-stage solutions act as a basis for the recourse actions, which have to satisfy the second-stage constraints in a cost-optimal way. To this end we have to guarantee that the transition from the first to the second stage is feasible. While the static constraints (7a) and (7c) need no further consideration, we neglect the possible impact of the constraints (7d). This is justified since the generation system is thermal dominated. The minimum up- and down-times constraints (7b) for the thermal units, however, need some refinement. In order to enforce compatibility between the first- and second-stage decisions we introduce constraints that relate the scheduling behaviours of the two stages to each other. This means that we prevent a thermal unit from being switched on or off in the second stage, if the scheduling history in the first stage prohibits that. The same canonical dependency is required in the other direction as well, i.e. we restrict switching in the first stage subject to the constraints set by the second-stage scheduling. Thus we have the constraints:

$$u_{it} - u_{i,t-1} \le 1 - (\mathbf{u}_{i,\sigma-1} - \mathbf{u}_{i,\sigma}),\ \sigma = t,\dots,\min\{t+\sigma_i-1,\ T\}, \tag{8a}$$

$$u_{i,t-1} - u_{it} \le 1 - (\mathbf{u}_{i,\tau} - \mathbf{u}_{i,\tau-1}),\ \tau = t,\dots,\min\{t+\tau_i-1,\ T\}, \tag{8b}$$

$$\mathbf{u}_{it} - \mathbf{u}_{i,t-1} \le 1 - (u_{i,\sigma-1} - u_{i,\sigma}),\ \sigma = t,\dots,\min\{t+\sigma_i-1,\ T\}, \tag{8c}$$

$$\mathbf{u}_{i,t-1} - \mathbf{u}_{it} \le 1 - (u_{i,\tau} - u_{i,\tau-1}),\ \tau = t,\dots,\min\{t+\tau_i-1,\ T\}, \tag{8d}$$

$$I\!P\text{ - a. s.},\ i = 1,\dots,I,\ t = 2,\dots,T-1.$$

Observe the consequences of the compatibility constraints (8). The inequality (8b), say, represents a constraint for the second stage if and only if unit i is switched off in the first stage at time t. In this case it enforces that the thermal unit will not be switched on in the second stage as long as the unit is cooling for its minimum down-time in the first stage. The remaining inequalities have similar effects.

Furthermore we introduce a subdivision of the set $\mathcal{I} := \{1, \dots, I\}$ of all thermal units into two subsets $\mathcal{I}_1$ and $\mathcal{I}_2$ such that $\mathcal{I}_1 \cup \mathcal{I}_2 = \mathcal{I}$ and the conditions $u_{it} = \mathbf{u}_{it}$, $i \in \mathcal{I}_2$, $t = 1, \dots, T$, $I\!\!P$ - a. s., are satisfied. This means that only some of the available thermal units may change their on/off state when compensating uncertain data. From a modelling point of view this approach leads to a reduction of the number of binary variables corresponding to a unit $i \in \mathcal{I}_2$. Moreover the case $\mathcal{I}_2 = \mathcal{I}$ conforms with the view taken in [3]. There all on/off decisions of the thermal units have been regarded as long-term decisions and thus belonging to the first stage only. Observe that (8) is clearly satisfied for all $i \in \mathcal{I}_2$.

The loading constraints (2) have to be adapted to the new situation. Here we distinguish between the two stages. As mentioned before we are looking for a solution to the here-and-now decisions that satisfies the uncertain demand with a certain probability and furthermore allows an optimal scheduling in each of the scenarios. That is why the first-stage power outputs of all generation units have to satisfy at least the expected load, while the second-stage power outputs are required to satisfy the load $\boldsymbol{d}_t$ with probability one. Hence the (modified) loading constraints are given by the following inequalities:

$$\sum_{i=1}^{I} p_{it} + \sum_{j=1}^{J} (s_{jt} - w_{jt}) \geq \mathbb{E}(\boldsymbol{d}_t), \ t = 1, \dots, T, \tag{9a}$$

$$\sum_{i=1}^{I} \mathbf{p}_{it} + \sum_{j=1}^{J} (\mathbf{s}_{jt} - \mathbf{w}_{jt}) \geq \boldsymbol{d}_t, \ t = 1, \dots, T, \ I\!\!P \text{ - a. s.} \tag{9b}$$

The reserve constraints (3) are modified in the same way. Note that from now on we use an equivalent characterization of the reserve constraints:

$$\sum_{i=1}^{I} u_{it}\, p_{it}^{\max} + \sum_{j=1}^{J} (s_{jt} - w_{jt}) \geq \mathbb{E}(\boldsymbol{d}_t + \boldsymbol{r}_t), \ t = 1, \dots, T, \tag{10a}$$

$$\sum_{i=1}^{I} \mathbf{u}_{it}\, p_{it}^{\max} + \sum_{j=1}^{J} (\mathbf{s}_{jt} - \mathbf{w}_{jt}) \geq \boldsymbol{d}_t + \boldsymbol{r}_t, \ t = 1, \dots, T, \ I\!\!P \text{ - a. s.} \tag{10b}$$

Again the second-stage decisions cover the specified amount with probability one, while the first-stage spinning reserve meets at least the expected demand. Finally we incorporate the stochastic fuel and electricity prices into the model. To this end we define the random variables $\boldsymbol{C}_{it}$, that describe the costs for operating thermal unit i in the second-stage during time period t, in the following way: $\boldsymbol{C}_{it}(p, u) := \max_{l=1,\dots,\bar{l}} \{\boldsymbol{a}_{ilt}\, p + \boldsymbol{b}_{ilt}\, u\}$, where $\boldsymbol{a}_{ilt}$, $\boldsymbol{b}_{ilt}$ are components of the random variable $\boldsymbol{\xi}_t$ that represent stochastic cost coefficients such that $\boldsymbol{C}_{it}(\cdot, 1)$ is $I\!P$-almost surely convex and increasing on $\mathbb{R}_+$. The cost functions $C_{it}(p, u)$ for the first stage we define accordingly, taking the expected values $\mathbb{E}(\boldsymbol{a}_{ilt})$, $\mathbb{E}(\boldsymbol{b}_{ilt}), l = 1, \ldots, \bar{l}$, as price coefficients, however. The effect of stochastic prices on the startup costs is modelled in a similar way. More precisely we have $\boldsymbol{S}_{it}(\mathbf{u}_i) := \mathbf{c}_{it}\left[\mathbf{u}_{it} - \mathbf{u}_{i,t-1}\right]^+$, $S_{it}(u_i) := \mathbb{E}(\boldsymbol{c}_{it})\left[u_{it} - u_{i,t-1}\right]^+$, where $\mathbf{c}_{it}$ are stochastic startup cost coefficients and $\mathbf{u}_i := (\mathbf{u}_{it})_{t=0}^T$, $\mathbf{u}_{i0} = u_{i0}$ $I\!P$ - a. s., $i = 1, \ldots, I$.

In consistency with common two-stage stochastic programming the objective function corresponds to the total costs for operating the thermal units in the first stage plus the expected costs in the second stage, i.e.,

$$\sum_{i=1}^{I} \sum_{t=1}^{T} \left[C_{it}(p_{it}, u_{it}) + S_{it}(u_i)\right] + \mathbb{E} \sum_{i=1}^{I} \sum_{t=1}^{T} \left[\boldsymbol{C}_{it}(\mathbf{p}_{it}, \mathbf{u}_{it}) + \boldsymbol{S}_{it}(\mathbf{u}_i)\right]. \quad (11)$$

The stochastic power production planning model consists then in minimizing the objective function (11) over all deterministic decisions (u, p, w, s) and all stochastic decisions $(\mathbf{u}, \mathbf{p}, \mathbf{s}, \mathbf{w}) \in L^\infty\left(\Omega, \mathcal{A}, I\!P; \mathbb{R}^{2T(I+J)}\right)$ satisfying the constraints (1), (4), (5), (7)–(10). The model represents a two-stage stochastic mixed-integer program involving $2(I+J)T$ deterministic and $2(I+J)T$ stochastic decision variables.

The stochastic program elaborated above is almost separable with respect to the units, since only the constraints (9)–(10) couple different units. This structure allows us to apply a stochastic version of the classical Lagrangian relaxation idea. We relax the above mentioned coupling constraints by introducing Lagrange multipliers $\boldsymbol{\lambda} := (\lambda^1, \lambda^2, \boldsymbol{\lambda}^3, \boldsymbol{\lambda}^4)$, where $\lambda^1, \lambda^2 \in \mathbb{R}_+^T$ and $\boldsymbol{\lambda}^3, \boldsymbol{\lambda}^4 \in L^1(\Omega, \mathcal{A}, I\!P; \mathbb{R}_+^T)$. Setting $x := (u, p, s, w)$ and $\mathbf{x} := (\mathbf{u}, \mathbf{p}, \mathbf{s}, \mathbf{w})$ the *Lagrangian* $L(x, \mathbf{x}; \boldsymbol{\lambda})$ is formed in the following way. Each of the constraints (9)–(10) is first associated with a Lagrange multiplier and then enters the objective function (11). Here the deterministic multipliers λ^1, λ^2 are linked with the first-stage constraints (9a) and (10a), whereas the stochastic variables $\boldsymbol{\lambda}^3, \boldsymbol{\lambda}^4$ are paired with the second-stage constraints (9b), (10b). Note that $L(\cdot)$ takes its values in $\mathbb{R}$, since the latter constraints enter the objective function via their expected value (cf. [10]). With the *dual function* $D(\boldsymbol{\lambda}) := \inf\limits_{(x, \mathbf{x})} L(x, \mathbf{x}; \boldsymbol{\lambda})$, where the infimum is taken subject to the constraints (1), (4), (5), (7) and (8), the *dual problem* reads

$$\max\left\{D(\boldsymbol{\lambda}) : \boldsymbol{\lambda} \in \mathbb{R}_+^{2T} \times L^1(\Omega, \mathcal{A}, I\!P; \mathbb{R}_+^{2T})\right\}. \quad (12)$$

The optimal value of the dual problem (12) provides a lower bound for the optimal costs of the (nonconvex) primal model. The minimization of L decomposes into stochastic single unit subproblems and the dual function

$$\begin{aligned} D(\boldsymbol{\lambda}) &= \sum_{i=1}^{I} D_i(\boldsymbol{\lambda}) + \sum_{j=1}^{J} \hat{D}_j(\boldsymbol{\lambda}) \\ &+ \sum_{t=1}^{T} \left[\lambda_t^1 \mathbb{E}(\boldsymbol{d}_t) + \lambda_t^2 \mathbb{E}(\boldsymbol{d}_t + \boldsymbol{r}_t) + \mathbb{E}\left(\boldsymbol{\lambda}_t^3 \boldsymbol{d}_t + \boldsymbol{\lambda}_t^4 (\boldsymbol{d}_t + \boldsymbol{r}_t)\right)\right] \end{aligned} \quad (13)$$

may be evaluated by solving thermal subproblems and hydro subproblems corresponding to $D_i(\boldsymbol{\lambda})$ and $\hat{D}_j(\boldsymbol{\lambda})$, respectively. The two kinds of subproblems represent two-stage stochastic programming models for the operation of a single unit. While the thermal subproblems are mixed-integer two-stage stochastic program, that reduce to combinatorial two-stage stochastic problems, the hydro subproblems are linear two-stage models. For the latter we make use of a descent algorithm described in [9], whereas for the thermal subproblems we employ stochastic dynamic programming when it is necessary. This means in particular that for the solution of the thermal subproblems we first relax the compatibility constraints (8) in order to apply a scenario decomposition approach. Then the stochastic dynamic programming need only be used if the preliminarily solution violates the compatibility constraints (cf. [10]). Extending Lagrangian relaxation approaches for deterministic power management models, our method for solving the stochastic two-stage model consists of the components described in Fig. 1. The non-smooth optimization problem (12) is solved with the proximal bundle method described in [7]. Bundle methods make use of function value and subgradient information in order to determine descent directions. The subproblem solvers provide that necessary information. The bundle method delivers an optimal value $D(\boldsymbol{\lambda}^*)$ of (12), which is a lower bound for the optimal cost of the primal model. In general, however, the dual optimal scheduling decisions violate the load and reserve constraints. With a Lagrangian heuristics, that takes its basic ideas from [5,12], we determine a primal feasible solution. For the case $\mathcal{I}_2 = \mathcal{I}$ we use a strategy that treats all scenarios simultaneously, whereas in all other instances each scenario is dealt with independently. Finally we fix the binary decisions and use a deterministic version of the economic dispatch algorithm in [9] for every scenario to find a nearly optimal primal solution. The interaction of the described components is illustrated in Fig. 1.

The stochastic Lagrangian relaxation algorithm was implemented in C++ except for the proximal bundle method, for which the FORTRAN-package NOA 3.0 [8] was used as a callable library. For testing the implementation a number of load scenarios was simulated from a time series model for the load process described in [6]. Furthermore the stochastic prices have been

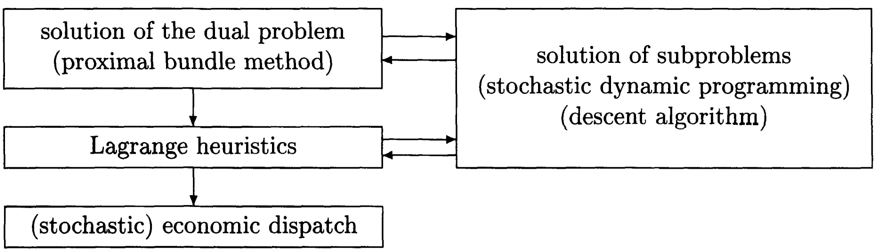

Fig. 1. Structure of the stochastic Lagrangian relaxation method

Scenarios	$\mathcal{I}_2$	opt. tol: 10^{-3} time [min] NOA	all	gap [%]	opt. tol: 10^{-4} time [min] NOA	all	gap [%]	obj.val [$\cdot 10^8$]
5	$\emptyset$	0:16	1:00	0.61	0:41	1:45	0.21	1.15511
5	$\emptyset$	0:18	0:35	0.25	0:45	1:18	0.17	1.49352
10	$\emptyset$	0:57	2:50	0.45	1:55	5:08	0.30	1.15563
10	$\emptyset$	0:44	2:04	0.31	1:32	3:53	0.13	1.42486
5	$\mathcal{I}$	0:10	0:28	0.90	0:26	0:43	0.70	1.50801
10	$\mathcal{I}$	0:27	0:59	1.25	1:14	1:43	0.87	1.44497
50	$\mathcal{I}$	9:00	12:04	1.83	12:36	16:07	1.36	1.41429
100	$\mathcal{I}$	30:17	35:31	1.99	35:37	42:48	1.70	1.43047

Table 1. Computing times and gaps (NOA 3.0: NGRAD= 20)

simulated by a discretized geometric Brownian motion. Test runs have been carried out for the weekly production planning (i.e. $T = 168$) of the hydro-thermal power generation system of VEAG comprising 25 thermal units and 7 pumped storage plants and for a number of scenarios ranging from 5 to 100. The corresponding primal optimization problems have up to 400.000 binary and 650.000 continuous variables, and more than 1.300.000 constraints. Table 1 shows computing times and gaps for different choices of the optimality

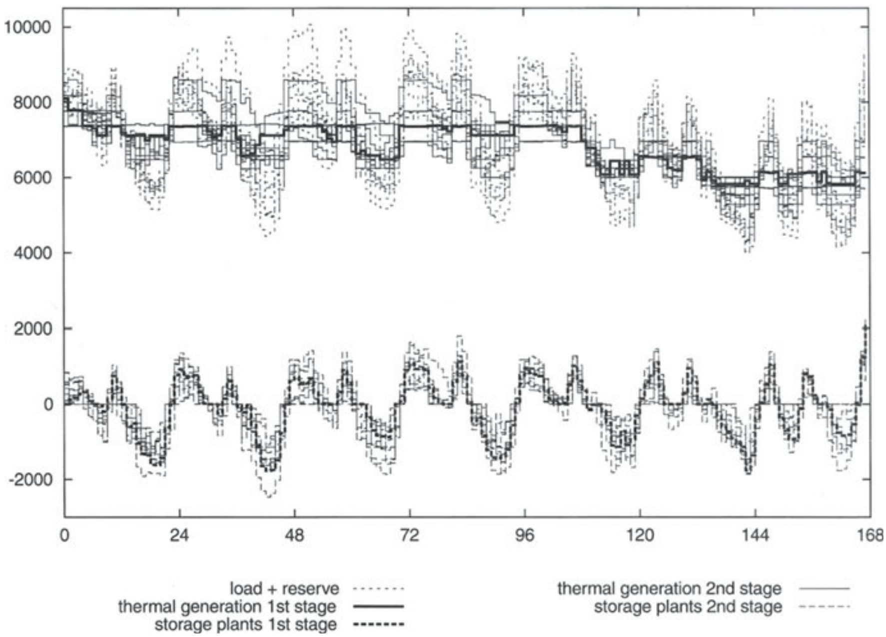

Fig. 2. Solution for 10 scenarios and $\mathcal{I}_2 = \emptyset$

tolerance for the proximal bundle method. The test runs have been performed on an HP 9000 (780/J280) Compute-Server with 180 MHz frequency and 768 MByte main memory under HP-UX 10.20. The results show that a smaller optimality tolerance leads to smaller gaps at the expense of the computing times. Here the gap refers to the relative difference of the costs of the determined scheduling decisions $(x, \mathbf{x})$ and the optimal value $D(\boldsymbol{\lambda}^*)$ of the dual problem. In the case of $\mathcal{I}_2 = \emptyset$ the performance of the algorithm is closely related to the efficiency of the thermal subproblem's solver. In particular it depends on how often the stochastic dynamic programming algorithm is used during the dual maximization. In fact the complexity of the involved memory structures increases very fast, so that problem instances with more than 10 scenarios cannot be handled so far. Figure 2 provides a sample output of the algorithm. It is worth mentioning that typical solutions exhibit both switching on and off decisions at the transition from the first to the second stage. Thus a solution for this case in general yields a better objective function value than the solution to the corresponding problem, where the index set $\mathcal{I}_2$ consists of all thermal units (cf. [10]).

4 Day-Ahead Trading and Power Production

Power exchanges with future and spot markets are novel instruments arising at liberalized electricity markets [1]. Trading at a power exchange is becoming a means of ever growing importance in the utility's total economic activity. Coordination of trading and power production is a key issue in this respect. In what follows we address simultaneous power production and day-ahead trading at a power exchange. The production component is modeled according to Sect. 2.

Day-ahead trading involves sealed selling and purchase bids for every individual hour of the day ahead. Each offer comprises volume and price. There is only one round of bidding and the market price is cleared by an independent operator such that the total exchange is maximized. Figure 3 depicts the price formation mechanism: selling and purchase offers are placed in (price-) ascending and descending orders, respectively, yielding two step curves where step length corresponds to volume and step height to price. The intersection of the curves determines both the market price and the total volume traded. Selling offers strictly below and purchase offers strictly above market price are executed completely. Vice versa, sellings strictly above as well as purchases strictly below do not become effective. Offers at market price in general are executed only partially, with specific splitting rules in case of multiple offers with identical price.

A simultaneous management of electricity production and day-ahead bidding thus has to take into account two main factors: the utility's possibilities to generate or consume a certain amount of power at a certain price at a certain time, and the competitors' bids for that very time period. The latter

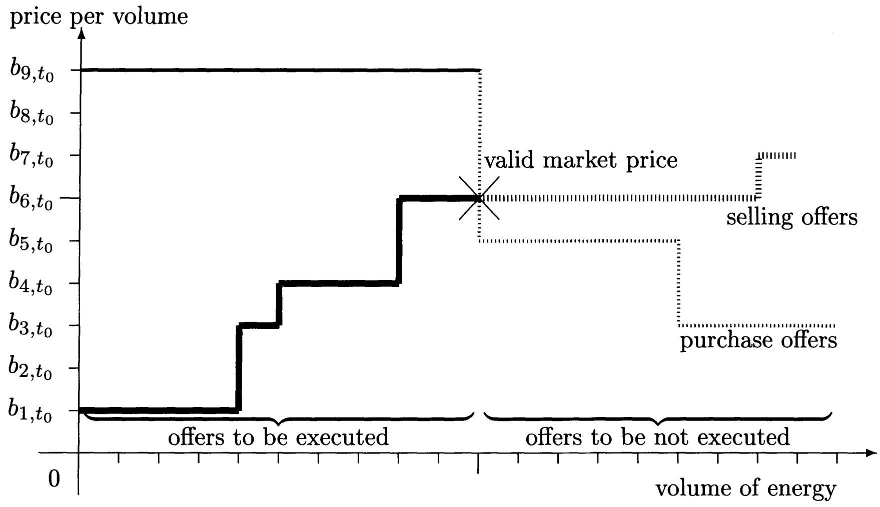

Fig. 3. Determining of the market price by mean of offers

being sealed and thus not available for the utility at the time of bidding, this induces considerable uncertainty for the utility at the moment of decision making.

The mathematical modelling of the phenomena sketched above is accomplished in two steps. First the price formation mechanism at the power exchange is formulated in mixed-integer linear terms and concatenated with the production model from Sect. 2. Afterwards, this deterministic optimization model is extended into a stochastic one by introducing probabilistic scenarios for the bidding behaviour of the competitors and heading for an optimization of the nonanticipative decisions to be taken by the utility. In what follows, we will display the first step in detail and only outline the extension towards stochastic programming.

To model price formation we consider the discretization $t = 1, \dots, T$ of the optimization horizon from Sect. 2. The price is discretized into $m = 1, \dots, M$ ascending levels with values $b_{mt} \geq 0$. For all $t = 1, \dots, T$, $m = 1, \dots, M$ we introduce triplets $(v^s_{mt}, v^p_{mt}, v^c_{mt}) \in \{0,1\}^3$ of indicators. The subsequent contraints system ensures that $v^s_{mt} = 1$ ($v^p_{mt} = 1$) iff the selling (purchase) offer with price level m at time t is strictly below (above) market price and hence executed completely. Moreover, we have $v^c_{mt} = 1$ iff price level m coincides with the valid market price at time t:

$$\begin{aligned} \sum_{m=1}^{M} v^c_{mt} = 1, \;\; t = 1, \dots, T; \\ \left. \begin{aligned} v^s_{mt} \geq v^s_{m+1,t}, \\ v^p_{mt} \leq v^p_{m+1,t}, \\ v^s_{mt} + v^c_{mt} + v^p_{mt} = 1, \end{aligned} \right\} \;\; t = 1, \dots, T, \; m = 1, \dots, M. \end{aligned} \tag{14}$$

We distinguish between variables for energy volumes of own offers of price level m in time period t (q^s_{mt}, $q^p_{mt} \geq 0$) and variables for volumes of own *executed* offers of price level m in time period t (p^s_{mt}, $p^p_{mt} \geq 0$). Executed offers cannot exceed offers:

$$0 \leq p^s_{mt} \leq q^s_{mt},\ 0 \leq p^p_{mt} \leq q^p_{mt}; \quad t = 1, \dots, T,\ m = 1, \dots, m. \tag{15}$$

Own selling offers below and own purchase offers above market price are to be executed completely:

$$\left.\begin{array}{l} p^s_{mt} \geq q^s_{mt} - C_1(1 - v^s_{mt}), \\ p^p_{mt} \geq q^p_{mt} - C_1(1 - v^p_{mt}), \end{array}\right\} \quad t = 1, \dots, T,\ m = 1, \dots, m \tag{16}$$

with a sufficiently big constant C_1. Own selling (purchase) offers must not be executed, if they are above (below) the valid price:

$$\left.\begin{array}{l} p^s_{mt} \leq C_2(v^s_{mt} + v^c_{mt}), \\ p^p_{mt} \leq C_2(v^p_{mt} + v^c_{mt}), \end{array}\right\} \quad t = 1, \dots, T,\ m = 1, \dots, m \tag{17}$$

with a sufficiently big constant C_2.

The competitors' (or foreign) selling and purchase offers of price level m for time t are denoted by $f^s_{mt}, f^p_{mt} \geq 0, m = 1, \dots, M, t = 1, \dots, T$, respectively. It is feasible that not the complete volume of a competitor's offer at market price is executed (as in our example, Fig. 3). β^s_t, $\beta^p_t \geq 0$ then denote the actually executed volumes of the competitors at valid market price:

$$0 \leq \beta^s_t \leq \sum_{m=1}^{M} v^c_{mt} f^s_{mt},\ 0 \leq \beta^p_t \leq \sum_{m=1}^{M} v^c_{mt} f^p_{mt}, \quad t = 1, \dots, T. \tag{18}$$

Maximum exchange is reached at the equilibrium of supply and demand:

$$\sum_{m=1}^{M} (v^s_{mt} f^s_{mt} + p^s_{mt}) + \beta^s_t = \sum_{m=1}^{M} (v^p_{mt} f^p_{mt} + p^p_{mt}) + \beta^p_t,\ \ t = 1, \dots, T. \tag{19}$$

At market price, either a complete selling offer or a complete purchase offer or both a complete selling and a complete purchase offer have to be executed (cf. Fig. 4). Introducing another indicator $v^0_t \in \{0, 1\}$ attaining the value 1 iff at least all selling offers at market price are executed, the price formation model is completed by the following constraints:

$$\begin{array}{l} \beta^s_t \geq \sum_{m=1}^{M} v^c_{mt} f^s_{mt} - C_3(1 - v^0_t),\ \beta^p_t \geq \sum_{m=1}^{M} v^c_{mt} f^p_{mt} - C_3 v^0_t, \quad t = 1, \dots, T; \\ \left.\begin{array}{l} p^s_{mt} \geq q^s_{mt} - C_3(2 - v^c_{mt} - v^0_t), \\ p^p_{mt} \geq q^p_{mt} - C_3(1 - v^c_{mt} + v^0_t), \end{array}\right\} \quad t = 1, \dots, T, m = 1, \dots, M \end{array} \tag{20}$$

with a sufficiently big constant C_3.

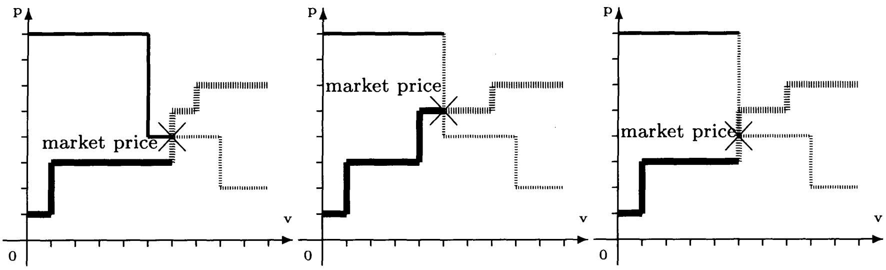

Fig. 4. Cases of price formation

Let $g_{mt} \in \mathbb{R}$ denote the net exchange at the power exchange of price level m at time t. It has to fulfill

$$\begin{aligned} \sum_{m=1}^{M} g_{mt} = \sum_{m=1}^{M} (p^p_{mt} - p^s_{mt}), \quad t = 1, \dots, T, \\ -C_4 v^c_{mt} \leq g_{mt} \leq C_4 v^c_{mt}, \quad t = 1, \dots, T, \; m = 1, \dots, M \end{aligned} \tag{21}$$

with a sufficiently big constant C_4. Finally, concatenation with the production model from Sect. 2 is achieved by modifying the load coverage constraints into

$$\sum_{i=1}^{I} p_{it} + \sum_{j=1}^{J} (s_{jt} - w_{jt}) + \sum_{m=1}^{M} g_{mt} = d_t, \quad t = 1, \dots, T \tag{22}$$

and adding the term

$$\sum_{t=1}^{T} \sum_{m=1}^{M} b_{mt} g_{mt}. \tag{23}$$

to the objective (6).

The above model was validated with VEAG data for the power system and market prices of the Amsterdam Power Exchange (APX) [1] for the power exchange. The VEAG power system comprises 17 coal and 8 gas fired thermal units as well as 7 pumped storage plants. Using APX market prices from the time period July to December 1999 we constructed hourly foreign offers of different prices and volumes. To study impacts of power trading to power production we formed test problems both with actual APX price levels (problem B) and with levels deviating form the APX prices by up to 30% (problem C). In addition the model was run as a pure productions model, without trading (problem A). Model sizes varied from 18 000 to 63 000 constraints and 13 000 (2 500 integer) to 35 000 (6 000 integer) variables. Numerical tests were performed on a SUN E450 ultra SPARC, 300 MHz, using

Prob.	Time	Best Solution	Lower Bound	Gap	Min. Saving	Volumes Selling	Purchase
A	0:20:14	46 287 933	46 287 933	0.00%	0.00%	0	0
B	2:27:59	46 185 297	44 892 456	2.80%	0.22%	11 200	2 890
C	1:15:49	45 767 961	45 242 528	1.15%	1.12%	40 900	1 570

Table 2. Calculations

CPLEX Linear Optimizer 6.5.1 (1999), whose algorithmic basis is cutting-plane enhanced LP-based branch-and-bound.

Table 2 reports solution times and optimality certificates (gaps) together with the impact of trading. For problems B and C gains of 0.22% and 1.12%, respectively, were achieved, compared with the pure-production instance A. Solution times behind these savings are quite substantial. However, savings of 0.14% and 0.48% were achieved for problems B and C after only 43 and 20 minutes of solution time, respectively. Trading activities (selling and purchase volumes) over time for problem C are displayed in Fig. 5. These initial numerical results indicate the potential of simultaneously optimizing power production and trading.

To remove the unrealistic model assumption of anticipating the competitors' offers the latter are considered to be stochastic, i.e., given scenario-wise with certain probabilities. Then a two-stage stochastic program becomes appropriate. The principal modelling follows the lines sketched in a different context in Sect. 3: We consider an optimization horizon of 7 days and assemble all the decisions to be taken at the first day into the first stage. These comprise production descisions (variabels $p_{it}, s_{jt}, w_{jt}, l_{jt}$ for suitable t and all i, j) and the utility's offers for the subsequent day (variables $q^s_{mt}, q^p_{mt}, g_{mt}$ for suitable t and all m). The remaining variables enter the second stage. The objective function is a sum of direct costs caused by the first-stage decisions and of expected future costs in the second stage. Solution methodology for this two-stage stochastic integer program rest on the dual decomposition method developed in [2]. Again a non-smooth concave dual maximization problem has

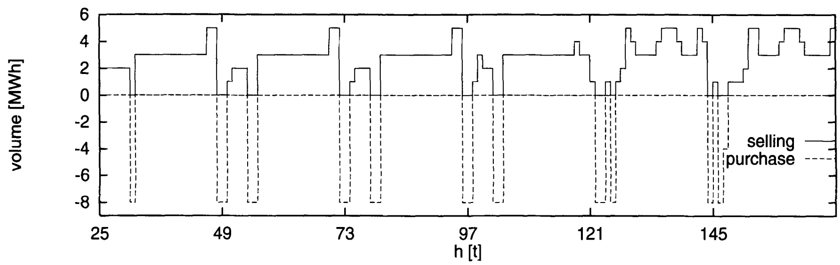

Fig. 5. Trading activities over time

to be solved (cf. Sect. 3) whose objective function values and subgradients are determined by solving single-scenario subproblems. The latter are of the same type as the above model (14)–(23). Research along these directions is still ongoing and will be reported elsewhere.

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