Mehrperiodische Risikofunktionale in der Energiewirtschaft

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Introduction

- Energiewirtschaftliche Modelle enthalten oft unsichere Parameter, die unabhängig von den Entscheidungen sind und für die (statistische) Daten existieren.
- Die unsicheren Daten können approximativ durch eine endliche Anzahl von Szenarien nebst deren Wahrscheinlichkeiten dargestellt werden.
- Die Szenarien besitzen Baumstruktur, falls ein Prozess rekursiver Beobachtungen und Entscheidungen abgebildet wird.
- Solche stochastischen Optimierungsmodelle besitzen Vorteile:
  - Entscheidungen sind robust gegenüber den Unsicherheiten.
  - Das Risiko von Entscheidungen kann modelliert und minimiert werden.

Ziel des Vortrages:

Risiko Modellierung und Minimierung

Ist $\{\xi_t\}_{t=0}^T$ ein stochastischer Prozess, der in ein energiewirtschaftliches Modell eingeht, $x = \{x_t\}_{t=0}^T$ ein stochastischer Entscheidungsprozess und beschreibt $f_t(x_t, \xi_t)$ den im Zeitpunkt t erzielten Ertrag, so hat der von der Entscheidung x abhängige Ertrag G_x über dem gesamten Zeithorizont die Form

$$G_x = \sum_{t=0}^{T} f_t(x_t, \xi_t).$$

G_x ist eine Zufallsvariable, deren Verteilung wesentlich von der Entscheidung x abhängt. Insbesondere kann G_x eine große Streuung besitzen und die Wahrscheinlichkeit

$$\mathbb{P}(G_x < \mathbb{E}(G_x)) \qquad \text{groß}$$

sein. Da dies kaum akzeptabel ist, ist eine nur auf die Maximierung des erwarteten Gesamtertrags gerichtete Entscheidung ungeeignet.

Deshalb wurden manchmal die (untere Semi-) Varianz oder der Value-at-Risk von G_x gleichzeitig minimiert.

Risk Functionals

Ein Risikofunktional ρ ordnet jeder Zufallsvariablen Y aus einem linearen (normierten) Raum $\mathcal{Y}$, z.B., $L_p(\Omega, \mathcal{F}, \mathbb{P})$ für ein $p \in [1, \infty]$, eine reelle Zahl zu. Solche Risikofunktionale ρ erfüllen folgende Axiome für alle Y, $\tilde{Y} \in \mathcal{Y}$, $r \in \mathbb{R}$, $\lambda \in [0, 1]$:

(A1) $\rho(Y + r) = \rho(Y) - r$ (Translations-antivarianz),

(A2) $\rho(\lambda Y + (1 - \lambda)\tilde{Y}) \leq \lambda\rho(Y) + (1 - \lambda)\rho(\tilde{Y})$ (Konvexität),

(A3) $Y \leq \tilde{Y}$ implies $\rho(Y) \geq \rho(\tilde{Y})$ (Monotonie).

Ein Risikofunktional ρ heißt kohärent, falls es überdies positiv homogen ist, d.h., $\rho(\lambda Y) = \lambda\rho(Y)$ für alle $\lambda \geq 0$ und alle $Y \in \mathcal{Y}$.

Für ein gegebenes Risikofunktional ρ heißt $\mathcal{D} = \mathbb{E} + \rho$ auch deviation Risikofunktional.

References: Artzner-Delbaen-Eber-Heath 99, Föllmer-Schied 02, Fritelli-Rosazza Gianin 02

Examples:

(a) (Conditional or) Average Value-at-Risk $\mathbb{AV@R}_\alpha$ ($\mathcal{Y} = L_1(\Omega)$):

$$
\begin{aligned}
\mathbb{AV@R}_\alpha(Y) &:= \frac{1}{\alpha}\int_0^\alpha \mathbb{V@R}_u(Y)(u)du \\
&= \inf\left\{x + \frac{1}{\alpha}\mathbb{E}([Y+x]^-) : x \in \mathbb{R}\right\} \\
&= \sup\left\{-\mathbb{E}(YZ) : \mathbb{E}(Z) = 1, 0 \le Z \le \frac{1}{\alpha}\right\}
\end{aligned}
$$

where $\alpha \in (0,1]$, $\mathbb{V@R}_\alpha := \inf\{y \in \mathbb{R} : \mathbb{P}(Y \le y) \ge \alpha\}$ is the Value-at-Risk and $[a]^- := -\min\{0, a\}$.

Reference: Rockafellar-Uryasev 02

(b) Lower semi standard deviation corrected expectation ($\mathcal{Y} = L_2(\Omega)$):

$$\rho(Y) := -\mathbb{E}(Y) + (\mathbb{E}([Y - \mathbb{E}(Y)]^-)^2)^{\frac{1}{2}}$$

Reference: Markowitz 52

Conditional risk mappings

Under certain assumptions there exists a conditional version of a risk functional ρ with respect to available information represented by a σ-field $\mathcal{G} \subset \mathcal{F}$:

$$\rho(Y|\mathcal{G})$$

as a mapping from $L_p(\mathcal{F})$ to $L_p(\mathcal{G})$.

Examples:

(a) Conditional Expectation

$$\rho(Y|\mathcal{G}) = -\mathbb{E}(Y|\mathcal{G})$$

(b) Conditional Average Value-at-Risk $\mathbb{A}\mathsf{V@R}_\alpha(Y|\mathcal{G})$

$$\mathbb{A}\mathsf{V@R}_\alpha(Y|\mathcal{G}) = \sup\left\{-\mathbb{E}(YZ|\mathcal{G}) : \mathbb{E}(Z|\mathcal{G}) = 1, 0 \le Z \le \frac{1}{\alpha}\right\}$$

Multi-Period Risk Functionals

Let $\xi = (\xi_1, \ldots, \xi_T)$ be an input random vector with associated filtration $\mathscr{F} = \{\mathcal{F}_t\}_{t=1}^T$. We assume that all decisions $Y = (Y_1, \ldots, Y_T)$ belong to $\mathcal{Y} := \times_{t=1}^T L_p(\Omega, \mathcal{F}, \mathbb{P})$ for some $p \in [1, \infty]$.
A functional ρ that associates to any pair $(Y, \mathscr{F})$, $Y \in \mathcal{Y}$ and $\mathscr{F}$ denoting a filtration, a real number $\rho(Y; \mathscr{F})$ is called multi-period risk functional if it satisfies for all Y and $\tilde{Y}$ in $\mathcal{Y}$ and filtrations $\mathscr{F}$:

(A1) $\rho(Y_1+W_1, \ldots, Y_T+W_T; \mathscr{F}) = -\sum_{t=1}^T \mathbb{E}(W_t) + \rho(Y_1, \ldots, Y_T; \mathscr{F})$ for all W belonging to some convex subset $\mathcal{W} = \mathcal{W}(\mathscr{F})$ in $\mathcal{Y}$ ($\mathcal{W}$-translation-antivariance),

(A2) $\rho(\,\cdot\,; \mathscr{F})$ is convex on $\mathcal{Y}$ (convexity),

(A3) $Y_t \le \tilde{Y}_t$, for all t, implies $\rho(Y_1, \ldots, Y_T; \mathscr{F}) \ge \rho(\tilde{Y}_1, \ldots, \tilde{Y}_T; \mathscr{F})$ (monotonicity).

The set $\mathcal{W}$ is related to the set of available (financial) instruments for hedging the risk.

References: Artzner-Delbaen-Eber-Heath-Ku 07, Fritelli-Scandolo 06, Pflug-Römisch 07

Example: (for the set $\mathcal{W}$)

(a) $\mathcal{W} = \{(x, 0, \ldots, 0) \in \mathbb{R}^T : x \in \mathbb{R}\} = \mathbb{R} \times \{0\}^{T-1}$

(Artzner-Delbaen-Eber-Heath-Ku 07).

(b) $\mathcal{W} = \mathbb{R}^T$.

(c) $\mathcal{W} = \{W = (W_1, \ldots, W_T) : \sum_{t=1}^{T} W_t$ is deterministic$\}$.

(Fritelli-Scandolo 06).

(d) $\mathcal{W} = \{W = (W_1, \ldots, W_T) : W_t$ is measurable w.r.t. $\mathcal{F}_{t-1}\}$

(Pflug-Ruszczynski 04).

Another **requirement**:

(A0) $\rho(Y; \mathscr{F}) \geq \rho(Y; \mathscr{F}')$ if $Y \in \mathcal{Y}$ and $\mathcal{F}_t \subseteq \mathcal{F}'_t$, $t = 1, \ldots, T$ (information monotonicity).

(Pflug-Römisch 07)

Polyhedral risk functionals:

Multi-period risk functionals are called polyhedral if they may be represented as optimal values of a linear stochastic program. Hence, they preserve linearity structures although such functionals are non-linear by nature.

(Eichhorn-Römisch 05)

Examples:

(a) Expectation of accumulated incomes $\sum_{\tau=1}^{t} Y_\tau$ at risk measuring time steps t_j, $j = 1, \ldots, J$, with $t_J = T$:

$$\rho_0(Y; \mathcal{F}) := -\sum_{j=1}^{J} \mathbb{E}\left(\sum_{t=1}^{t_j} Y_t\right)$$

(b) Sum of Average Value-at-Risk's at risk measuring time steps:

$$\rho_1(Y; \mathcal{F}) := \frac{1}{J}\sum_{j=1}^{J} \mathbb{A}\mathrm{V@R}_\alpha\left(\sum_{t=1}^{t_j} Y_t\right)$$

(c) Conditional Average Value-at-Risk compositions:

$$\rho_2(Y;\mathscr{F}) := \mathbb{AV@R}_\alpha(\cdot|\mathcal{F}_0) \circ \cdots \circ \mathbb{AV@R}_\alpha(\cdot|\mathcal{F}_{t_{J-1}})\Big(\sum_{j=1}^{T} Y_t\Big)$$

where $\mathcal{F}_0 = \{\emptyset, \Omega\}$. Does not satisfy (A0) and is not polyhedral!

(d) Sum of conditional Average Value-at-Risk's:

$$\rho_3(Y;\mathscr{F}) := \sum_{j=1}^{J} \mathbb{E}\Big(\mathbb{AV@R}_\alpha\Big(\sum_{t=1}^{t_j} Y_t \Big| \mathcal{F}_{t_{j-1}}\Big)\Big)$$

(e) Average Value-at-Risk of the average:

$$\rho_4(Y;\mathscr{F}) := \mathbb{AV@R}_\alpha\Big(\frac{1}{J}\sum_{j=1}^{J}\sum_{t=1}^{t_j} Y_t\Big)$$

(f) Average Value-at-Risk of the minimum:

$$\rho_6(Y;\mathscr{F}) := \mathbb{AV@R}_\alpha\Big(\min_{j=1,\ldots,J}\sum_{t=1}^{t_j} Y_t\Big)$$

Examples (b), (e), (f) are polyhedral risk functionals and satisfy (A1) with $\mathcal{W} = \mathbb{R} \times \{0\}^{J-1}$ and (d) is polyhedral.

Stochastic programming problem with risk objective:

$$\min_{x} \left\{ \rho(Y_1, \ldots, Y_T) \;\middle|\; \begin{array}{l} Y_t = \langle b_t(\xi_t), x_t \rangle, \\ x_t = x_t(\xi_1, \ldots, \xi_t) \in X_t, \\ \sum_{\tau=0}^{t-1} A_{t,\tau}(\xi_t) x_{t-\tau} = h_t(\xi_t) \\ (t = 1, \ldots, T) \end{array} \right\}$$

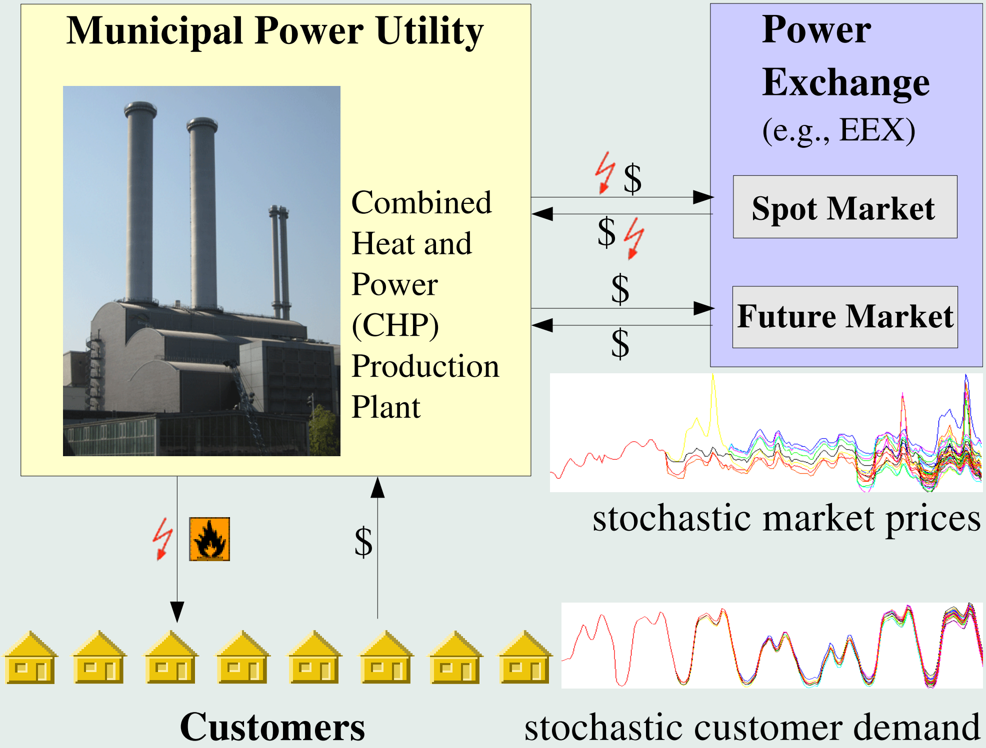
Polyhedral risk functional (evaluated at risk measuring time steps):

$$\rho(Y) = \inf \left\{ \mathbb{E}\left(\sum_{j=0}^{J} \langle c_j, v_j \rangle \right) \;\middle|\; \begin{array}{l} v_j = v_j(\xi_1, \ldots, \xi_{t_j}) \in V_j, \\ \sum_{k=0}^{j} B_{j,k} v_{j-k} = r_j \\ (j = 0, \ldots, J), \\ \sum_{k=0}^{j} \langle a_{j,k}, v_{j-k} \rangle = \sum_{t=1}^{t_j} Y_t \\ (j = 1, \ldots, J) \end{array} \right\}$$

Equivalent linear stochastic programming model:

$$\min_{(v,x)} \left\{ \mathbb{E}\left(\sum_{j=0}^{J} \langle c_j, v_j \rangle \right) \;\middle|\; \begin{array}{l} x_t = x_t(\xi_1, \ldots, \xi_t) \in X_t, \\ v_j = v_j(\xi_1, \ldots, \xi_{t_j}) \in V_j, \\ \sum_{s=0}^{t-1} A_{t,s}(\xi_t) x_{t-s} = h_t(\xi_t), \\ \sum_{k=0}^{j} B_{j,k} v_{j-k} = r_j, \\ \sum_{k=0}^{j} \langle a_{j,k}, v_{j-k} \rangle = \sum_{t=1}^{t_j} \langle b_t(\xi_t), x_t \rangle \end{array} \right\}$$

Mean-Risk Electricity Portfolio Management



We consider the electricity portfolio management of a German municipal electric power company. Its portfolio consists of the following positions:

- power production (based on company-owned thermal units),
- bilateral contracts,
- (physical) (day-ahead) spot market trading (e.g., European Energy Exchange (EEX)) and
- (financial) trading of futures.

The time horizon is discretized into hourly intervals. The underlying stochasticity consists in a multivariate stochastic load and price process that is approximately represented by a finite number of scenarios. The objective is to **maximize the total expected revenue and to minimize the risk**. The portfolio management model is a large scale (mixed-integer) multi-stage stochastic program.

Electricity portfolio management: statistical models and scenario trees

For the stochastic input data of the optimization model (here yearly electricity and heat demand, and electricity spot prices), a statistical model is employed. It is calibrated to historical data in the following way:

- cluster classification for the intra-day (demand and price) profiles,

- 3-dimensional time series model for the daily average values (deterministic trend functions, a trivariate ARMA model for the (stationary) residual time series),

- simulation of an arbitrary number of three dimensional sample paths (scenarios) by sampling the white noise processes within the ARMA model and by adding on the trend functions and matched intra-day profiles from the clusters afterwards,

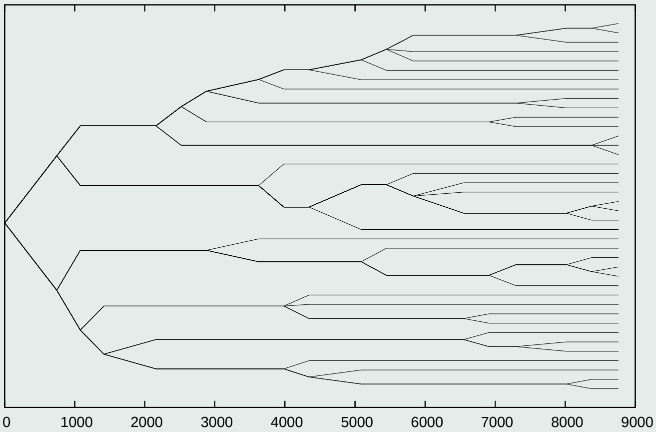
- generation of scenario trees (Heitsch-Römisch 09).

Electricity portfolio management: Results

Test runs were performed on data of a German municipal power company leading to a linear program containing $T = 365{\cdot}24 = 8760$ time steps, a scenario tree with 40 demand-price scenarios (see below) with about 150.000 nodes. The objective function is of the form

$$\text{Minimize} \quad \gamma\rho(Y) - (1-\gamma)\mathbb{E}\left(\sum\nolimits_{t=1}^{T} Y_t\right)$$

with a (multi-period) risk functional ρ with risk aversion parameter $\gamma \in [0,1]$ ($\gamma = 0$ corresponds to the risk-neutral case).



Single-period and multi-period risk functionals are computed for the accumulated income at $t = T$ and at the risk time steps t_j, $j = 1, \ldots, J = 52$, respectively. The latter correspond to 11 pm at the last trading day of each week.

It turns out that the numerical results for the expected maximal revenue and minimal risk

$$\mathbb{E}\left(\sum_{t=1}^{T} Y_t^{\gamma *}\right) \qquad \text{and} \qquad \rho(Y_{t_1}^{\gamma *}, \ldots, Y_{t_J}^{\gamma *})$$

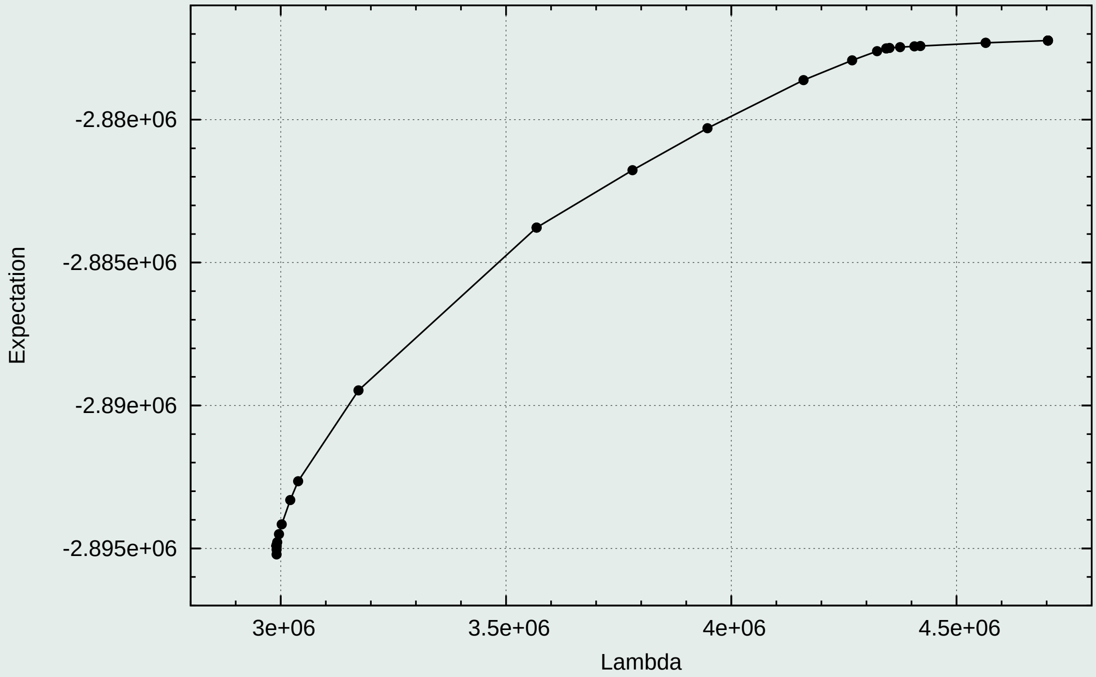
with the optimal income process $Y^{\gamma *}$ are **identical** for $\gamma \in [0.15, 0.95]$ and all risk functionals used in the test runs.

The efficient frontier

$$\gamma \mapsto \left(\rho(Y_{t_1}^{\gamma *}, \ldots, Y_{t_J}^{\gamma *}), \mathbb{E}\left(\sum_{t=1}^{T} Y_t^{\gamma *}\right)\right)$$

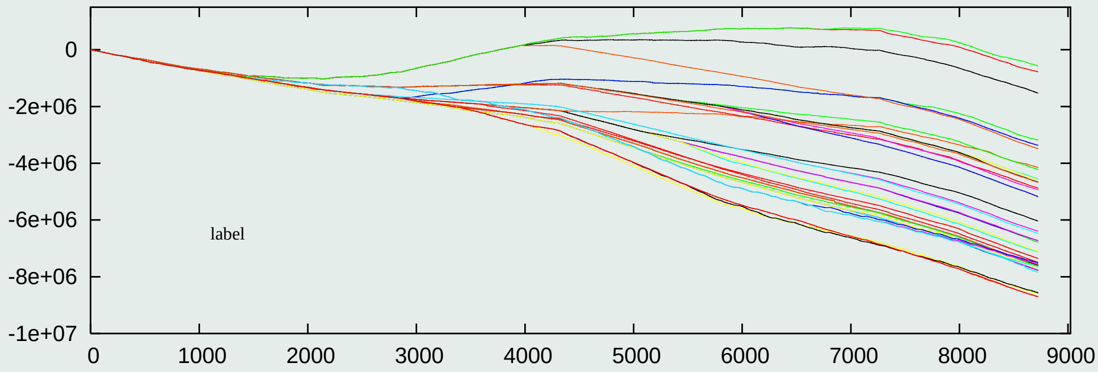
is concave for $\gamma \in [0, 1]$.

Risk aversion costs less than 1% of the expected overall revenue.

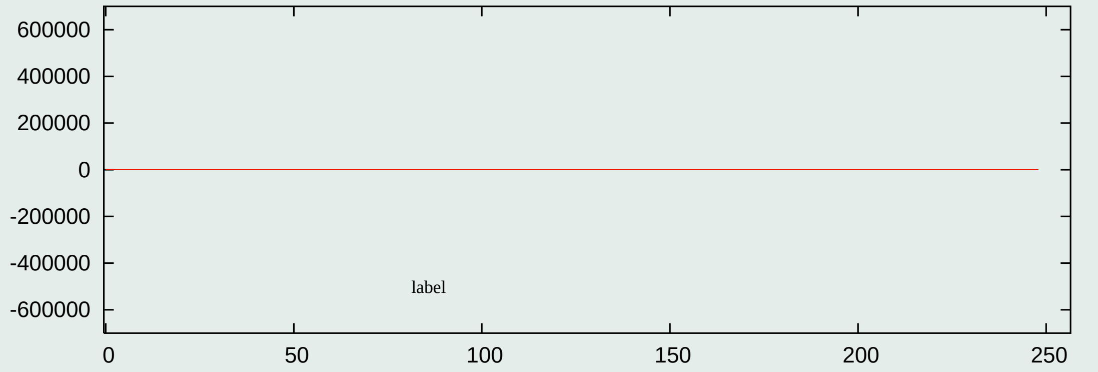


Efficient frontier

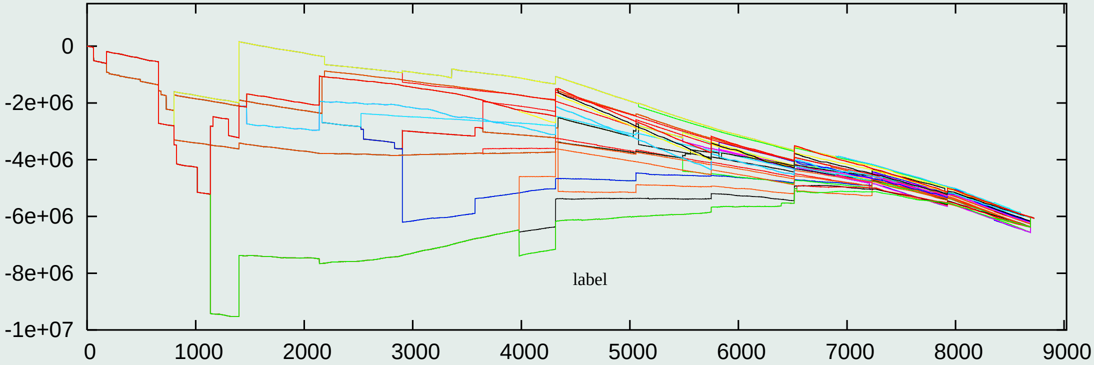
The LP is solved by CPLEX 9.1 in about 1 h running time on a 2 GHz Linux PC with 1 GB RAM.



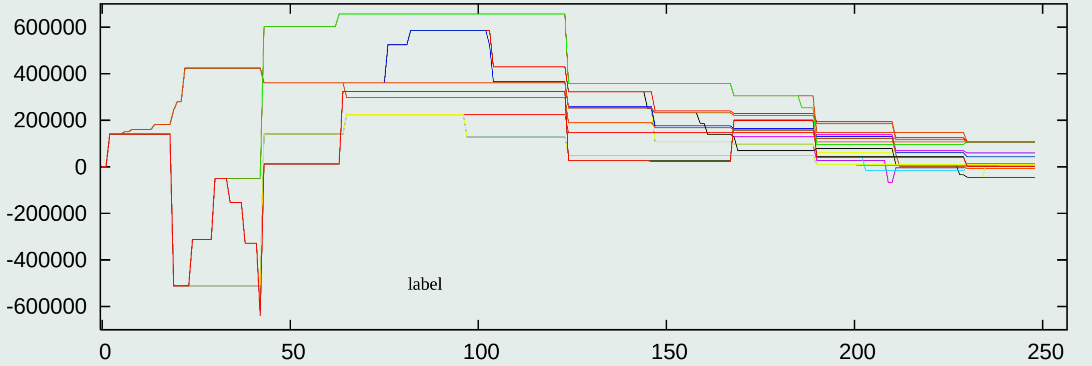
Overall revenue scenarios for $\gamma = 0$



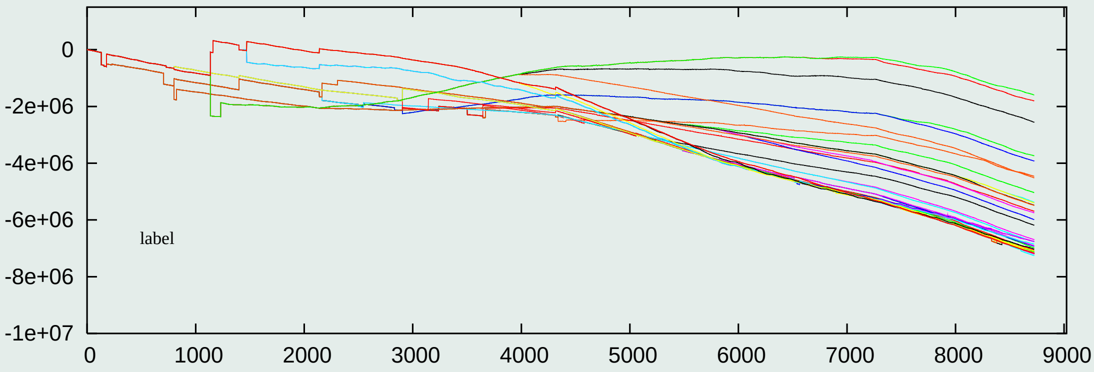
Future trading for $\gamma = 0$



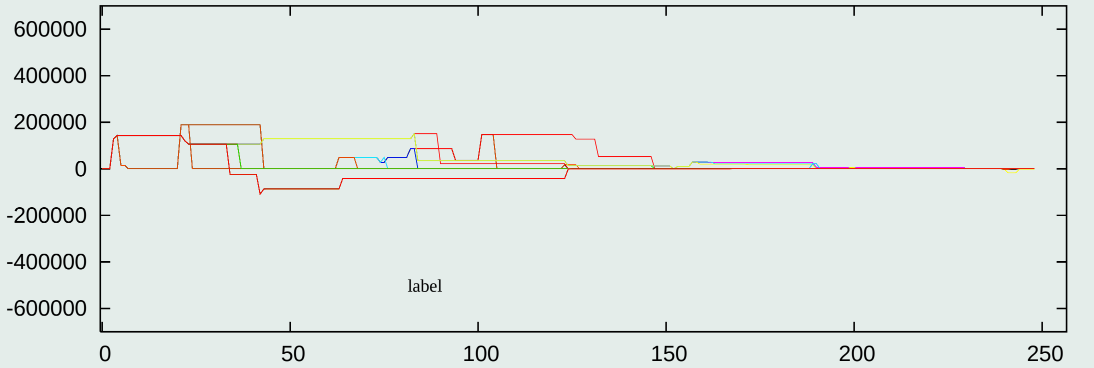
Overall revenue scenarios with $\mathbb{A}$V@R$_{0.05}$ and $\gamma = 0.9$



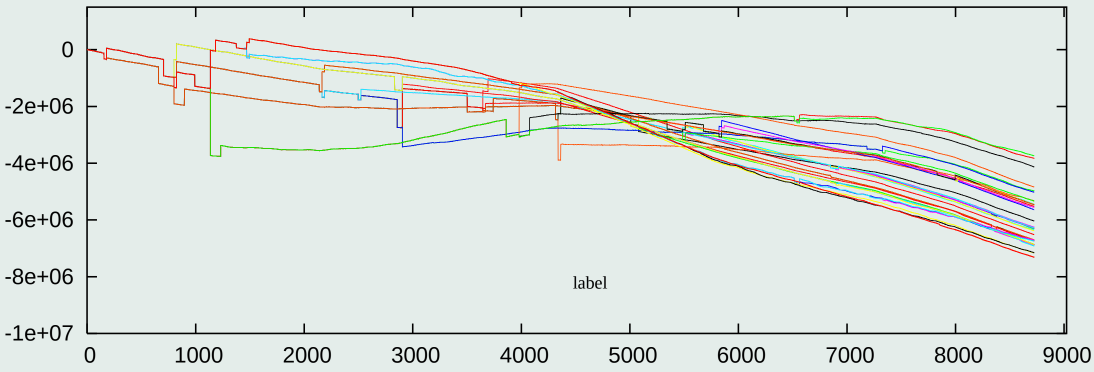
Future trading with $\mathbb{A}$V@R$_{0.05}$ and $\gamma = 0.9$



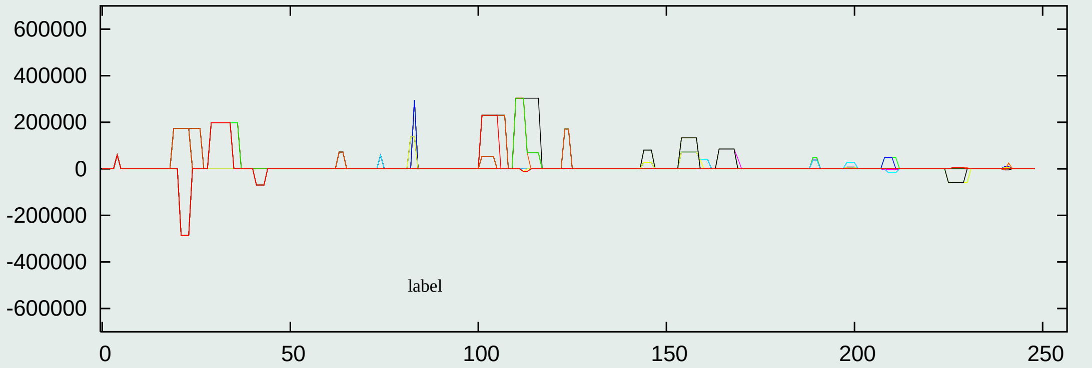
Overall revenue scenarios with ρ_1 and $\gamma = 0.9$



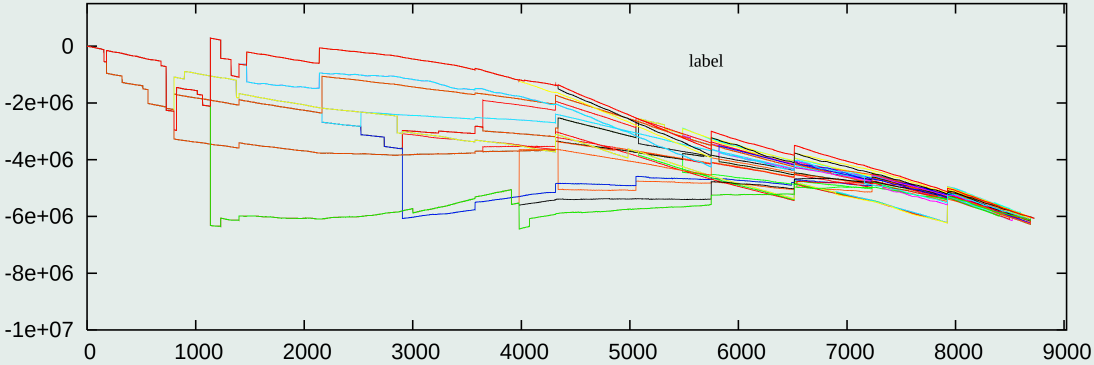
Future trading for ρ_1 and $\gamma = 0.9$



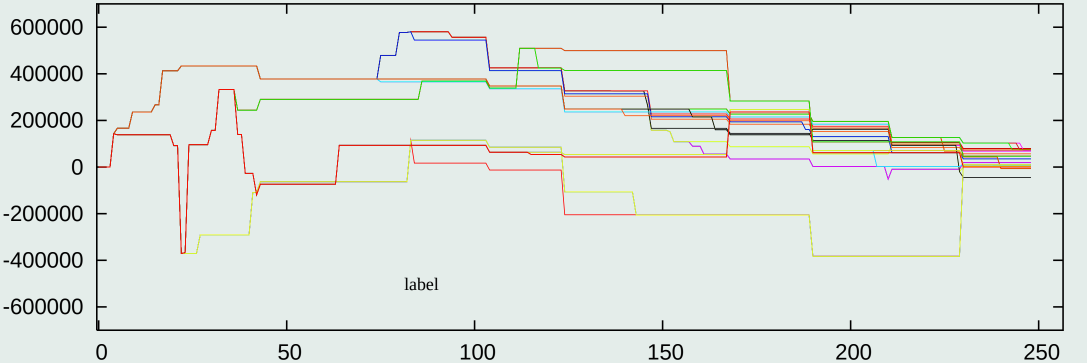
Overall revenue scenarios with ρ_4 and $\gamma = 0.9$



Future trading with ρ_4 and $\gamma = 0.9$



Overall revenue scenarios with ρ_6 and $\gamma = 0.9$



Future trading with ρ_6 and $\gamma = 0.9$

Thank you

References:

Eichhorn, A., Heitsch, H., Römisch, W.: Stochastic optimization of electricity portfolios: Scenario tree modeling and risk management, in *Handbook of Power Systems* (P.M. Pardalos, M. Pereira et al Eds.), Springer, 2009 (erscheint).

Eichhorn, A., and Römisch, W.: Dynamisches Risikomanagement in der Energiewirtschaft, *Innovative Modellierung und Optimierung von Energiesystemen* (R. Schultz, H.-J. Wagner Hg.), LIT Verlag, 2009, 255–271.

Pflug, G. Ch.; Römisch, W.: *Modeling, Measuring and Managing Risk*, World Scientific, Singapore, 2007.

Pflug, G. Ch.; Römisch, W.: The role of information in multi-period risk measurement, *Stochastic Programming E-Print Series* 5 (2009) (www.stopro.org).

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