

## OBITUARY MANFRED BREUER

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Manfred Breuer, an influential mathematician and inspiring teacher, died on January 31, 2011, in Marburg, Germany, where he had been full professor of mathematics from 1971 till 1996. Born in 1929, he attended a German high school that was located in an area under French occupation after World War II and that had to adopt the French schooling system. Out of this experience blossomed his lifelong active interest in French language, science and culture. Breuer studied mathematics in Mainz and later in Bonn, where he got his PhD in 1957, supervised by Wolfgang Krull. His thesis [3] on Jacobian differential systems contains developments in the direction of modern symplectic geometry and Poincaré manifolds. He was Krull's assistant between 1957 and 1959, when he became fascinated with von Neumann algebras (vNAs in what follows) through Dixmier's book [14]. Breuer was then invited to Berkeley twice, in 1959 - 1961 and in 1963 - 1965. There he became acquainted with Fredholm theory in collaboration with Heinz-Otto Cordes; his Habilitationsschrift of 1965 [6] arose from these new perspectives. From 1966 to 1971, the most productive period of his career, he was full professor at the University of Kansas in Lawrence. There he wrote his most influential work, on the Fredholm theory of semifinite vNAs [8, 7]. In 1969, Atiyah invited him to Oxford where he also met Singer; both were interested in Breuer's work for different reasons, while his work took new directions under their influence, as seen in his paper on bundles with vNA fibers [9].

Breuer's collaboration with Cordes [4, 5] emphasized homotopy theoretic arguments in Banach algebras, somehow anticipating the K-theory of operator algebras which was fully developed only much later. In his work on Fredholm theory in semifinite vNAs, he introduces the 'Breuer index' which takes values in the 'index group'. In modern terms, this is nothing but  $K_0$ . By choosing a faithful trace, one obtains a real valued index. In Atiyah's influential  $L^2$ -index theorem [1] a faithful trace arises naturally from the underlying geometry while in Breuer's more general

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approach no trace is preferred over any other. There are many developments arising from Atiyah's result such as  $L^2$ -Reidemeister-Franz torsion [10] or  $L^2$ -analytic torsion [15, 16] which can be seen in hindsight to have been influenced by Breuer's work.

In parallel with Breuer's work, but completely independently, the theory of operator ideals in semi-finite vNAs was developed. Both lines of research coalesced in the late 1990's in the study of semifinite non-commutative geometry; this, in turn, led to renewed interest in Breuer's work. The impetus for extending Connes' framework in the context of semifinite vNAs was only partly due to the influence of Atiyah's  $L^2$ -index theorem, since Connes and Cuntz [13] it had been demonstrated that the study of cyclic cohomology leads naturally to semifinite Fredholm modules. The systematic study of semifinite non-commutative geometry was begun only in 1998 in [11], and in connection with foliation theory (à la Connes) by Benamèur and Fack [2]. The motivation for [11] was provided by Phillips' general theory of the analytic spectral flow [17] which depends on the Breuer index. Breuer's approach was indeed surprisingly prescient because, with only minor modifications, it can be adapted to the situation of the local index formula in semifinite noncommutative geometry, even though a complete account had to wait until 2006 [12].

In his Marburg period, Breuer's research interests gradually reduced to a few questions. His later years were focussed on the proof of one of Kaplanski's conjectures which says that  $AW^*$ -factors are in fact  $W^*$ -factors. Breuer's attempts at employing a homotopy theoretical argument to prove the existence of a trace in  $AW^*$ -factors, unfortunately, did not succeed. But his interest in new developments in mathematics did not fade away, they were studied in lectures and seminars well beyond the date of his retirement. With similar energy he tried to perfect his astonishing knowledge of cultural history, with special emphasis on its French sector.

Those who met Manfred Breuer in person will remember him as a penetrating and far sighted researcher, as a helpful and inspiring teacher, and as a gracious and self-effacing human being.

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