Discontinuous Galerkin Finite Flement Method, Theory and Applications to Computational Fluid Dynamics

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The discontinuous Galerkin finite element method (DGFEM) is a nonconforming finite element technique using piecewise polynomial approximations of a solution of initial-boundary value problems without any requirement on the continuity of approximate solutions on interfaces between neighbouring elements. This allows to derive in a natural way sufficiently accurate and robust numerical schemes for the numerical treatment of problems with solutions containing boundary and internal layers or discontinuities. From this point of view, the DGFEM is suitable for the solution of linear or nonlinear convection-diffusion problems with dominating convection and the solution of compressible flow, where shock waves, contact discontinuities and boundary layers have to be resolved.

1) **Basic concepts from the discontinuous Galerkin finite element metod**: properties of FE meshes, function spaces, conforming and nonconforming methods, multiplicative trace inequality, inverse inequality, derivation of the DGFEM for a model elliptic problem, IIPG, NIPG and SIPG versions of the discretization.

2) **Analysis of the DGFEM for an elliptic problem**: coercivity of NIPG, IIPG and SIPG versions, error estimate in H^1-seminorm.

3) **Optimal error estimate in L^2-norm for an elliptic problem**: Application of the Nitsche duality trick to the DGFEM, examples, experimental order of convergence, comparison of theoretical and computational results.

4) **DGFEM in space for a model scalar nonlinear nonstationary convection-diffusion equation**: continuous problem, space DG semidiscretization of an initial-boundary value problem, numerical flux, numerical realization of the discrete problem.

5) **Error estimates of the DG space semidiscretization for a nonlinear nonstationary convection-diffusion problem**: coercivity of NIPG, IIPG and SIPG versions, error estimate in H^1-seminorm, optimal error estimate in L^2-norm, numerical experiments.

6) **Space-time DG discretization of convection-diffusion problems**: space DG semidiscretization, time DG discretization, space-time DG interpolation and its approximation properties, analysis of error estimates.

7) **DGFEM for the solution of compressible flow I**:
basic equations of gas dynamics, DG discretization of the Euler equations describing inviscid flow, numerical flux, explicit and semi-implicit schemes, some important ingredients: realization of boundary conditions, isoparametric elements, artificial viscosity in the vicinity of discontinuities (shock waves and contact discontinuities) for avoiding the Gibbs phenomenon.

8) **DGFEM for the solution of compressible flow II**:
DG for compressible viscous flow, discretization of convection and viscous terms, partial linearization, DG-ALE method for flows in time-dependent domains, examples.

Language of the course: English

Recommended literature:

V. Dolejší, M. Feistauer: Error estimates of the discontinuous Galerkin method for nonlinear nonstationary convection-diffusion problems. Numerical Functional Analysis and Optimization, 26 (2005), 349-383.

M. Feistauer, V. Kučera: On a robust discontinuous Galerkin technique for the solution of compressible flow. J. Comput. Phys. 224 (2007), 208-221.

M. Feistauer, J. Felcman, I. Straškraba: Mathematical and Computational Methods for Compressible Flow. Oxford University Press, Oxford, 2003, ISBN 0 19 850588 4

Previous knowledge expected:
differential and integral calculus, Green's theorem, basic knowledge of numerical methods

Lecture website: http://www.mathematik.hu-berlin.de/~ccafm/dG-FEM-Lecture/