MINLP for ECS Optimization

A brief description of the Problem

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Goal

Develop a Mathematic Tool
for the Economic and Thermodynamic
Optimal Design of a EC Plant

Energy Conversion (EC)

An example

$$CH_4 + 2O_2 \longrightarrow CO_2 + 2H_2O_{vapor}.$$

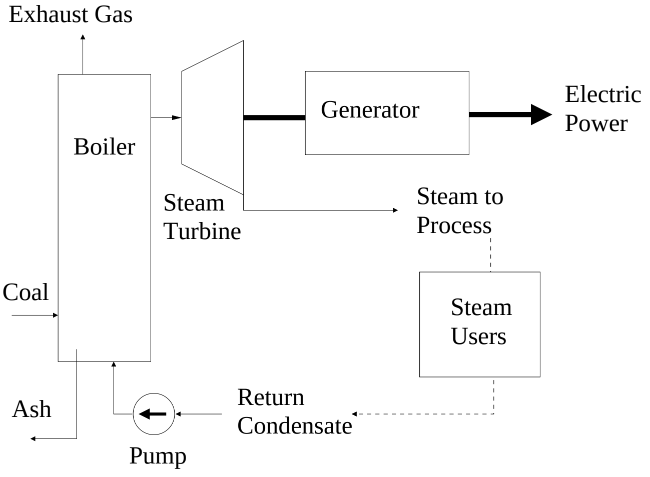
A cogeneration plant

Chemical Energy transformed into
Electric Power + Steam

An Example

Cogenerator Design

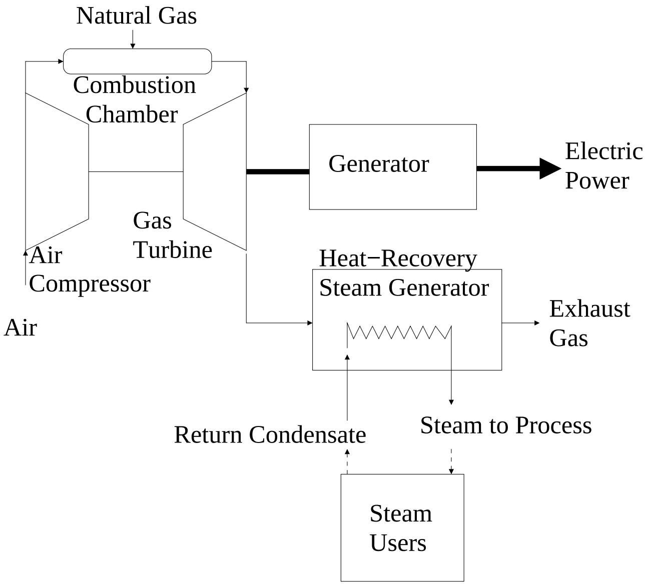
Option 1: Coal-fired steam turbine



An Example

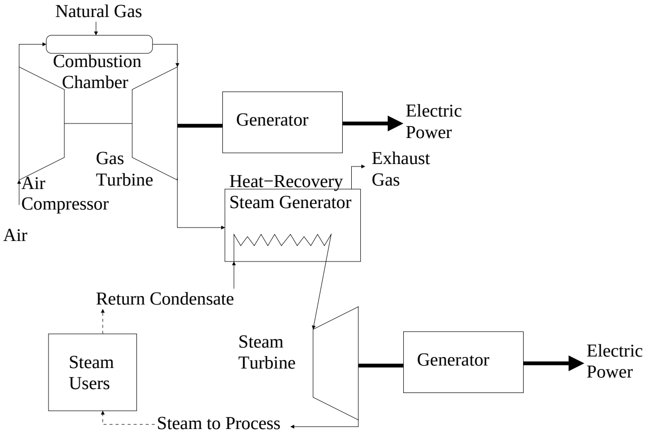
Cogenerator Design

Option 2: Gas turbine

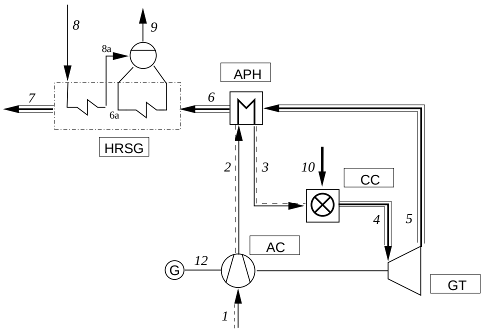


An Example
Cogenerator Design

Option 3: Gas and steam turbine



Consider one Design Option



AC	Air Compressor
APH	Air PreHeater
CC	Combustion Chamber
GT	Gas Turbine
HRSG	Heat Recovery Steam Generator

Modeling the Problem with the constraints

Example: Combustion Chamber

$$\dot{n}_4 = \dot{n}_3 + \dot{n}_{10} \qquad \text{(flow conservation)}$$

$$\dot{Q}_v = \dot{n}_3 h_3(T_3) + \dot{n}_{10} h_{10}(T_{10}) - \dot{n}_4 h_4(T_4) \qquad \text{(energy conservation)}$$

$$\dot{Q}_v = 0.02\mathrm{LHV}\dot{n}_{10} \qquad \text{(heat loss)}$$

$$\dot{n}_4 x_{4,\mathrm{N_2}} = 0.79\dot{n}_3 \qquad \text{(nitrogen fraction)}$$

$$\dot{n}_4 x_{4,\mathrm{O_2}} = 0.21\dot{n}_3 - 2\dot{n}_{10} \qquad \text{(oxygen fraction)}$$

$$\dot{n}_4 x_{4,\mathrm{CO_2}} = \dot{n}_{10} \qquad \text{(carbon dioxide fraction)}$$

$$\dot{n}_4 x_{4,\mathrm{H_2O}} = 2\dot{n}_{10} \qquad \text{(vapor fraction)}$$

$$x_{4,\mathrm{CH_4}} = 0 \qquad \text{(methane fraction)}$$

Variables

$\dot{n}_i$	flow at point i [mol/s]
T_i	temperature at point i [K]
p_i	pressure at point i [bar]
$x_{i,j}$	percentage composition of element j at position i
h_i	specific enthalpy of the flow i
s_i	specific entropy of the flow i
η_k	efficiency of component k
$\dot{W}_i$	power generated at point i
$\dot{Q}_v$	Heat Loss
PEC_k	cost of component k
	other economic variables
	superstructural binary variables

Objective

min sum of levelized costs

Constraints

- Flow conservation
- Energy Conservation (enthalpy)
- Efficiency property (entropy)
- Equipment purchase cost functions
- Economic variables
- Superstructural

Some of the Nonlinear Constraints

$$
\begin{aligned}
\mathrm{PEC}_{\mathrm{ac}} &= \left(\frac{C_{11}\dot{n}_a}{C_{12}-\eta_{sc}}\right)\left(\frac{p_2}{p_1}\right)\log\left(\frac{p_2}{p_1}\right) \\
\mathrm{PEC}_{\mathrm{cc}} &= \left(\frac{C_{21}\dot{n}_a}{C_{22}-\frac{p_4}{p_3}}\right)\left(1+e^{C_{23}T_4-C_{24}}\right) \\
h_i(T) &= \sum_{j\in F_i} x_{i,j}\Big(H_j^{+}+a_jT+\frac{b_j}{2}T^2 \\
&\quad -c_jT^{-1}+\frac{d_j}{3}T^3\Big) \\
s_i(T,p) &= \sum_{j\in F_i} x_{i,j}\Big(S_j^{+}+a_j\ln T+b_jT \\
&\quad -\frac{c_j}{2}T^{-2}+\frac{d_j}{2}T^2-R\ln\frac{x_jp}{p_{\mathrm{ref}}}\Big) \\
\eta_{sc} &= \frac{h_{2s}(T_{2s})-h_1(T_1)}{h_2(T_2)-h_1(T_1)}
\end{aligned}
$$

Some of the Other Constraints

Flow conservation

$$\dot{n}_4 = \dot{n}_3 + \dot{n}_{10}$$

Energy Conservation

$$\dot{W}_1 = \dot{n}_2 h_2(T_2) - \dot{n}_1 h_1(T_1) \geq 0$$

Power requirement

$$-100\ \mathrm{kW} \leq -(\dot{W}_2 + \dot{W}_1) - 30\,000\ \mathrm{kW} \leq 100\ \mathrm{kW}$$

Progress

- formulated the example design in AMPL[*] for efficiency optimization
- solved using SNOPT[†] via the NEOS server

Current state of the Project

- working on the superstructure formulation
- development of LaGO

[*] A Mathematical Programming Language

[†] Sparse Nonlinear OPTimization