# Algebraic Geometry 2018, Problem Set Nr. 6, Prof. Gavril Farkas, HU Berlin

1. Show that for $m > n$ there does not exist a nonconstant regular map

$$\phi : \mathbb{P}^m \to \mathbb{P}^n.$$

2. Let $\Sigma = \sigma(\mathbb{P}^2 \times \mathbb{P}^2) \subset eq\mathbb{P}^8$ be the Segre variety of dimension 4. By arguments similar to Exercise 1, show that the secant (chordal) variety $S(\Sigma)$ of $\Sigma$ has dimension 7.

3. Let $V$ be a vector space of dimension $n+1$ over $k$. Show that an element $\omega \in \wedge^2 V$ is decomposable if and only if $\omega \wedge \omega = 0$. Conclude once more that the Grassmannian $G(2, V) \subset \mathbb{P}(\wedge^2 V)$ is a projective variety cut out by quadratic relations.

4. Let $p \in \mathbb{P}^3$ be a point and $H \subset \mathbb{P}^3$ a plane containing $p$. We denote by $\Sigma \subset \mathbb{G}(1,3)$ the subvariety of the Grassmannian of consisting of those lines $l \subset H$ such that $p \in l$. Show that under the Plücker embedding $\mathbb{G}(1,3) \subset \mathbb{P}^5$, the subvariety $\Sigma$ is carried to a line. Conversely, show that every line in $\mathbb{P}^5$ lying on $\mathbb{G}(1,3)$ appears in this way.

5. Let $l_1, l_2 \subset \mathbb{P}^3$ be skew lines. Show that the locus $Q \subset \mathbb{G}(1,3)$ of lines in $\mathbb{P}^3$ meeting both $l_1$ and $l_2$ is the intersection of $\mathbb{G}(1,3)$ with a suitable 3-plane $\mathbb{P}^3 \subset \mathbb{P}^5$. Deduce once more that $Q$ is isomorphic to $\mathbb{P}^1 \times \mathbb{P}^1$. What happens if the lines $l_1$ and $l_2$ meet?

6. Let $S := S_{k,\ell} \subseteq \mathbb{P}^{k+\ell+1}$ be a scroll with $k, \ell \geq 2$. Show that the projection from one of the lines of the ruling induces a biregular isomorphism of $S_{k,\ell}$ and $S_{k-1,\ell-1} \subseteq \mathbb{P}^{k+\ell-1}$.

7. Show that any $k+1$ lines of the ruling of a scroll $S_{k,\ell} \subseteq \mathbb{P}^{k+\ell+1}$ ($k \leq \ell$) are independent, that is, span a $\mathbb{P}^{2k+1}$), but any $k+2$ lines are dependent, that is, are contained in a $\mathbb{P}^{2k+2}$.