Blatt 13

Aufgabe 1

$$\begin{cases} 5x + 4z + 2u = 3 \\ x - y + 2z + u = 1 \\ 4x + y + 2z = 1 \\ x + y + z + u = 0 \end{cases} \Longleftrightarrow \begin{pmatrix} 5 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 4 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ u \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\left(\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right) \xrightarrow{z_4 \leftrightarrow z_1} \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 5 & 0 & 4 & 2 & 3 \end{array}\right) \xrightarrow[\substack{z_3 - 4z_1 \\ z_4 - 5z_1}]{z_2 - z_1}$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 & 1 \\ 0 & -3 & -2 & -4 & 1 \\ 0 & -5 & -1 & -3 & 3 \end{array}\right) \xrightarrow[z_4 - z_3 - z_2]{} \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 & 1 \\ 0 & -3 & -2 & -4 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right) \xrightarrow{-z_3}$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 & 1 \\ 0 & 3 & 2 & 4 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right) \xrightarrow{z_3 + \frac{3}{2} z_2} \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 & 1 \\ 0 & 0 & \frac{7}{2} & 4 & \frac{1}{2} \\ 0 & 0 & 0 & 1 & 1 \end{array}\right)$$

$$\Rightarrow \quad u = 1$$

$$z = \left(\frac{1}{2} - 4\right) \cdot \frac{2}{7} = \frac{1-8}{2} \cdot \frac{2}{7} = -1$$

$$y = (1 - z) \cdot \frac{1}{-2} = 2 \cdot \frac{1}{-2} = -1$$

$$x = 0 - u - z - y = -1 + 1 + 1 = 1$$

Aufgabe 2

$$A = \begin{pmatrix} t+7s & -t+5s & 4t-2s \\ 5t-6s & 2t-4s & -t+6s \\ 3t-3s & t-s & 2s \end{pmatrix} \quad \text{über } \mathbb{R}$$

$$\xrightarrow[S_2+S_3]{S_1+S_3} \begin{pmatrix} 5t+5s & 3t+3s & 4t-2s \\ 4t & t+2s & -t+6s \\ 3t-s & t+s & 2s \end{pmatrix} \xrightarrow{S_1-S_2} \begin{pmatrix} 2t+2s & 3t+3s & 4t-2s \\ 3t-2s & t+2s & -t+6s \\ 2t-2s & t+s & 2s \end{pmatrix}$$

$$\xrightarrow[z_2-z_3]{z_1+z_3} \begin{pmatrix} 4t & 4t+4s & 4t \\ t & s & -t+4s \\ 2t-2s & t+s & 2s \end{pmatrix} \xrightarrow{\frac{1}{4}z_1} \begin{pmatrix} t & t+s & t \\ t & s & -t+4s \\ 2t-2s & t+s & 2s \end{pmatrix} \xrightarrow{z_3-z_1}$$

$$\longrightarrow \begin{pmatrix} t & t+s & t \\ t & s & -t+s \\ t-2s & 0 & -t+2s \end{pmatrix} \xrightarrow{S_1+S_3} \begin{pmatrix} 2t & t+s & t \\ s & s & -t+s \\ 0 & 0 & -t+2s \end{pmatrix}$$

$$\xrightarrow{z_2-z_3} \begin{pmatrix} 2t & t+s & t \\ s & s & -s \\ 0 & 0 & -t+2s \end{pmatrix} \xrightarrow{z_1+z_3} \begin{pmatrix} 2t & t+s & 2s \\ s & s & -s \\ 0 & 0 & -t+2s \end{pmatrix}$$

$$\xrightarrow{S_3+S_1-2S_2} \begin{pmatrix} 2t & t+s & 0 \\ s & s & -2s \\ 0 & 0 & -t+2s \end{pmatrix} \xrightarrow{S_1-S_2} \begin{pmatrix} t-s & t+s & 0 \\ 0 & s & -2s \\ 0 & 0 & -t+2s \end{pmatrix}$$

$s = 0 \Rightarrow A' = \begin{pmatrix} t & t & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -t \end{pmatrix} \Rightarrow \begin{array}{l} t = 0 \Rightarrow \text{rang } A = 0 \\ t \neq 0 \Rightarrow \text{rang } A = 1 \end{array}$

$s \neq 0$: $-t + 2s = 0 \Rightarrow t = 2s \Rightarrow$

$$A' = \begin{pmatrix} s & 3s & 0 \\ 0 & s & -2s \\ 0 & 0 & 0 \end{pmatrix}$$

$$\downarrow$$

$$\begin{pmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \text{rang } A = 2$$

$-t + 2s \neq 0$: $t - s = t + s = 0 \Rightarrow t = 0$

$$\Rightarrow A' = \begin{pmatrix} 0 & 0 & 0 \\ 0 & s & -2s \\ 0 & 0 & -t+2s \end{pmatrix} \Rightarrow \text{rang } A = 2$$

$t - s \neq 0$ oder $t - s \neq 0 \Rightarrow \text{rang } A = 3$

Aufgabe 3

$$A = \begin{pmatrix} -1 & 0 & 1 \\ 1-i & 0 & 2 \\ 1 & -2 & -1 \end{pmatrix}$$

$$\det A = \det \begin{pmatrix} -1 & 0 & 1 \\ 1-i & 0 & 2 \\ 0 & -2 & 0 \end{pmatrix} = \det \begin{pmatrix} 0 & 0 & 1 \\ 3-i & 0 & 2 \\ 0 & -2 & 0 \end{pmatrix} =$$

$$= 1 \left(-2(3-i) - 0 \right) = -6 + 2i$$

$$B = \begin{pmatrix} 1 & 0 & 2i & 0 \\ 0 & 2i & 2i & 2i \\ 2i & 2 & -1 & 0 \\ 2i & 0 & 0 & 2i \end{pmatrix}$$

$$\det B = \det \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2i & 2i & 2i \\ 2i & 2 & 3 & 0 \\ 2i & 0 & 4 & 2i \end{pmatrix} = \det \begin{pmatrix} 2i & 2i & 2i \\ 2 & 3 & 0 \\ 0 & 4 & 2i \end{pmatrix}$$

$$= (2i) \det \begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 0 \\ 0 & 4 & 2i \end{pmatrix} = 4i \det \begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 0 \\ 0 & 2 & i \end{pmatrix}$$

$$= 4i \det \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ 0 & 2 & i \end{pmatrix} = 4i \det \begin{pmatrix} 1 & -2 \\ 2 & i \end{pmatrix} = 4i(i+4)$$

$$= -4 + 16i$$

$$C = \begin{pmatrix} \bar{2} & \bar{2} & \bar{3} \\ \bar{3} & \bar{1} & \bar{4} \\ \bar{2} & \bar{3} & \bar{2} \end{pmatrix} \qquad \text{über } \mathbb{Z}/5\mathbb{Z}$$

$$\det C \xlongequal[S_2+S_3]{S_1+S_3} \det \begin{pmatrix} \bar{0} & \bar{0} & \bar{3} \\ \bar{2} & \bar{0} & \bar{4} \\ \bar{4} & \bar{0} & \bar{2} \end{pmatrix} = \bar{3} \det \begin{pmatrix} \bar{2} & \bar{0} \\ \bar{4} & \bar{0} \end{pmatrix} = \bar{0}.$$