

The Skorokhod embedding problem: old and new

Abstract: The Skorokhod embedding problem (SEP) is to represent a given probability measure as the distribution of Brownian motion at a chosen stopping time. Over the last 50 years the SEP has become one of the important classical problems in probability theory with a huge variety of different solutions that have been employed in various branches of pure and applied probability. In recent years there has been a revived interest in the SEP due to its connection to model independent finance and martingale optimal transport.

In the first lecture, I will give three recent applications of specific solutions to the SEP, a proof of the Brascamp Lieb inequality, a counterexample to the Cantelli conjecture, and model independent upper bounds for exotic options. In the second lecture, I will show an approach to the SEP based on ideas of optimal transport that allows the systematic construction of solutions to the SEP with particular optimality properties. Moreover, it turns out that this approach directly extends to sufficiently regular Markov processes. In the last lecture I will present extensions to multiple marginals with applications in model independent finance and the PCOC problem of finding martingales with a continuum of prescribed marginals.