"Bounds on Value-at-Risk with Partial Dependence Information"

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We study the problem of finding bounds on Value-at-Risk (VaR) in the presence of model uncertainty. In particular we consider a portfolio, consisting of d risks $X_1, ..., X_d$ whose individual distributions are known whereas only partial information on the dependence structure between the constituents is available. This is in the literature often referred to as (partial) dependence uncertainty. In this context, a large part of the academic research has focused on the computation or estimation of bounds on the VaR of the sum $\sum_{i=1}^{d} X_i$ in the absence of information on the dependence structure. These bounds however tend to be impractically wide as they omit all information on the dependence between the risks. Therefore, methods to estimate bounds on the aggregate VaR which account for additional information about the interaction between the individual risks are becoming increasingly popular. We account for several types of additional dependence information in the computation of VaR bounds. Firstly, we obtain bounds in the case where the joint law of $(X_1, ..., X_d)$ is known on a compact subset of $\mathbb{R}^d$. This generalizes the setting in Bernard and Vanduffel (2015) such as to integrate information on the lower-dimensional marginals of the joint distribution. Secondly, we consider information about the distance between the joint distribution of $(X_1, ..., X_d)$ and a given reference distribution with respect to a distance on the set of distributions. The solutions are based on an improvement of the Fréchet-Hoeffding bounds which account for the partial information on the dependence structure. By an extension of the results of Embrechts and Puccetti (2006) we are then able to translate the improved Fréchet-Hoeffding bounds to estimates for the maximal and minimal VaR.