Some Applications of Multicriterion Strategies in Mechanical Engineering

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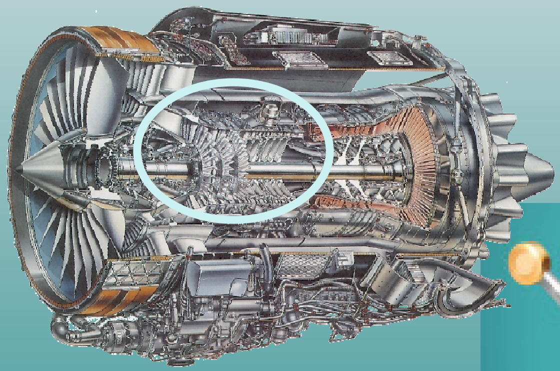
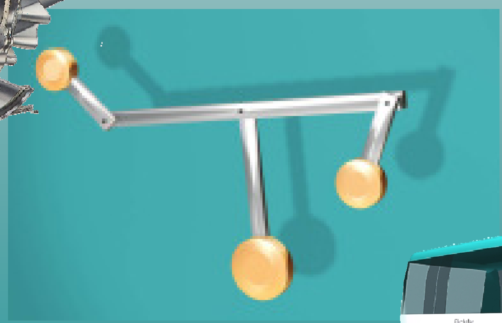
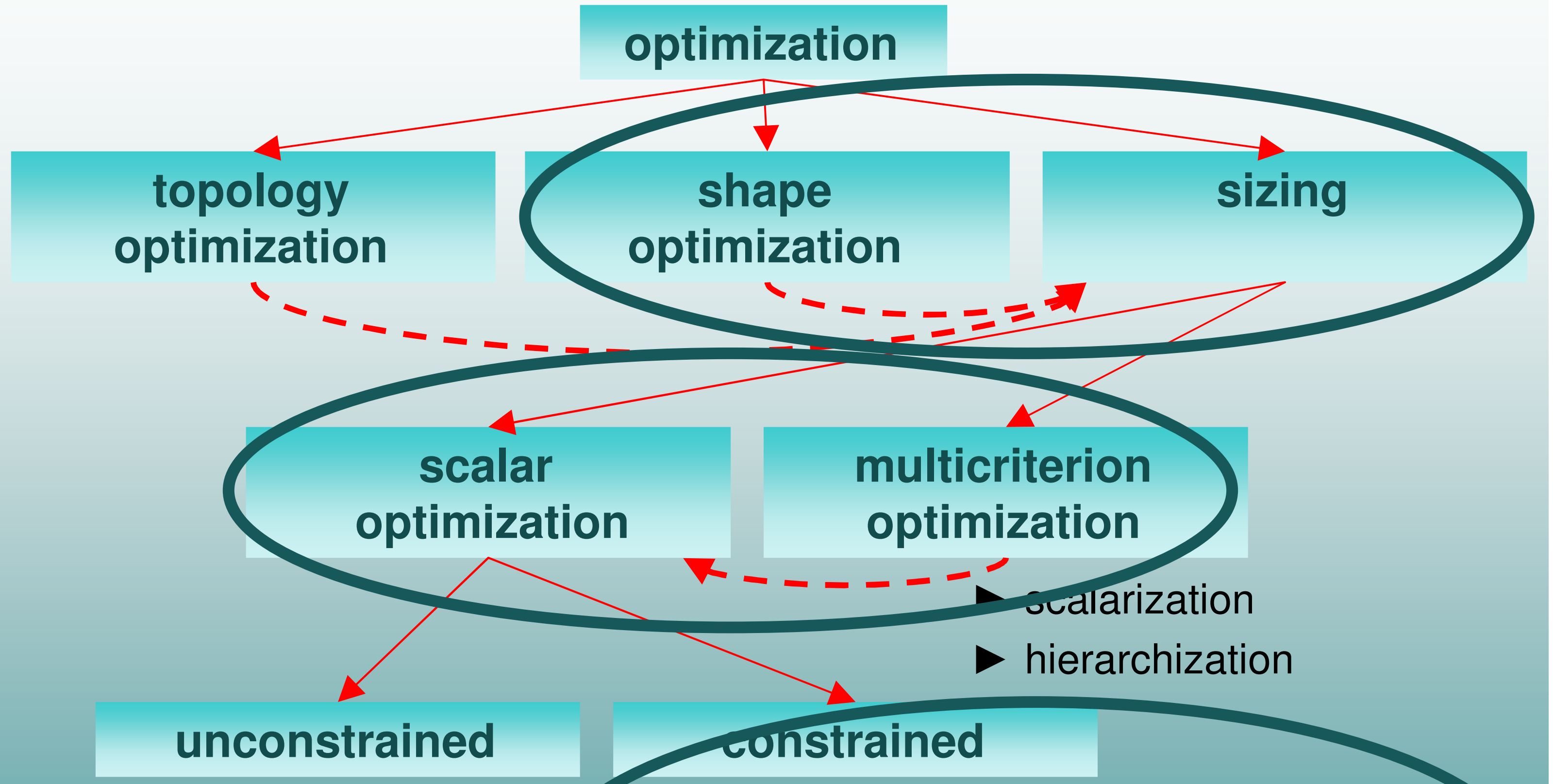




Classification of Optimization Problems



$$\min_{\mathbf{p}\in \mathbb{R}^h} f(\mathbf{p})$$

$$\min_{\mathbf{p}\in P} f(\mathbf{p})$$

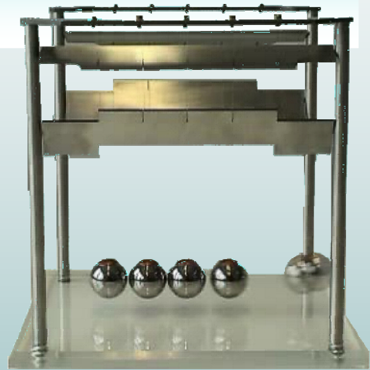
$$P = \left\{ \mathbf{p} \in \mathbb{R}^h \,\middle|\, \mathbf{g}(\mathbf{p}) = \mathbf{0},\ \mathbf{h}(\mathbf{p}) \le \mathbf{0},\ \mathbf{p}^u \le \mathbf{p} \le \mathbf{p}^o \right\}$$

What is Optimization Good For?

- **Designing Birthday Presents**

$$\min_{\gamma,\,\Delta x,\,\varepsilon} \; \max_{t} \; \{|x_2(t)|, |x_3(t)|, |x_4(t)|\}$$

minimum motion of inner balls



$$\min_{\rho(\varphi)} \; T$$

$$s.t. \quad s(T) = \hat{s}$$

minimum time

$$\min_{\mathbf{p} \in P} \begin{bmatrix} \Omega^l \\ \Omega_0 - \Omega^u \end{bmatrix}$$

maximum stable speed range

$$\Omega^l := \min_{0 \le \Omega \le \Omega_0} \Omega \qquad s.t. \; \max_j \mathrm{Re}(\lambda_j) \le 0$$

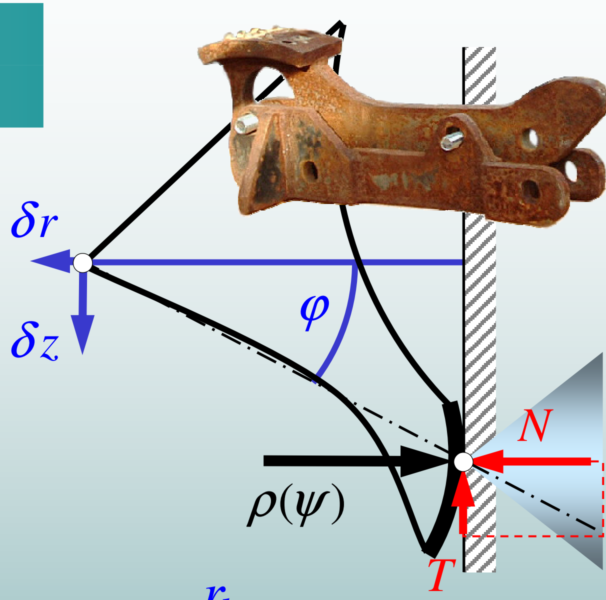
$$\Omega^u := \min_{\Omega^l < \Omega \le \Omega_0} \Omega \qquad s.t. \; \max_j \mathrm{Re}(\lambda_j) > 0$$

What is Optimization Good For?

- Designing Birthday Presents
- Solving Problems of Existing Machines

$$\begin{bmatrix} \max\limits_{\rho(\psi)} & \delta r \\ \min\limits_{\rho(\psi)} & \delta z \end{bmatrix} \quad s.t. \quad \tan\varphi \le \mu_0$$

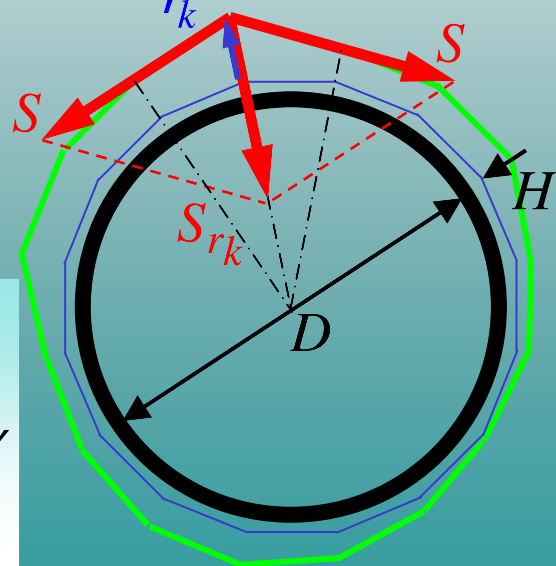
ideal contact geometry



$$\min_{\mathbf{r},S,\gamma} \; \gamma + w\,\mathbf{r}^T\mathbf{r}$$

$$s.t. \;\; S^0_{r_k} = G_k / \mu_0 \, , \quad \frac{\left| S_{r_k} - S^0_{r_k} \right|}{S^0_{r_k}} \le \gamma$$

ideal ring shape

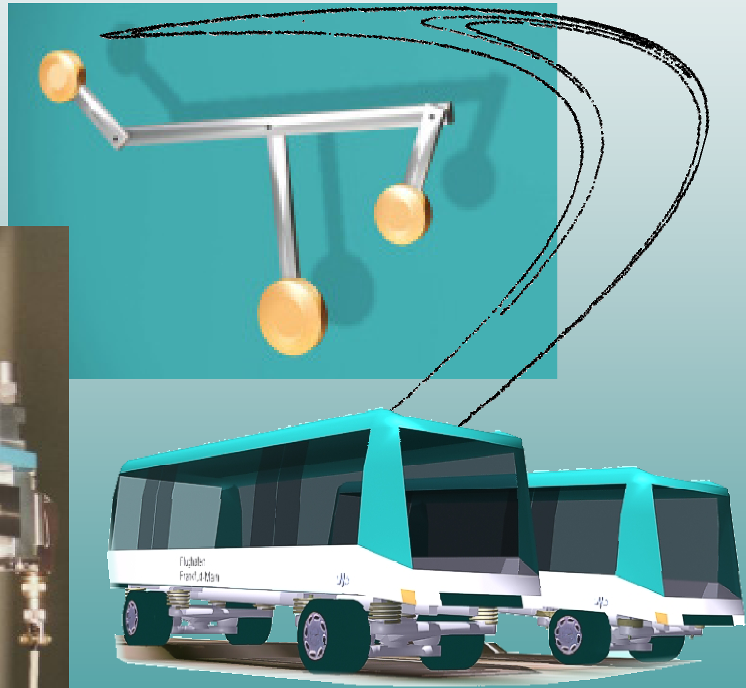


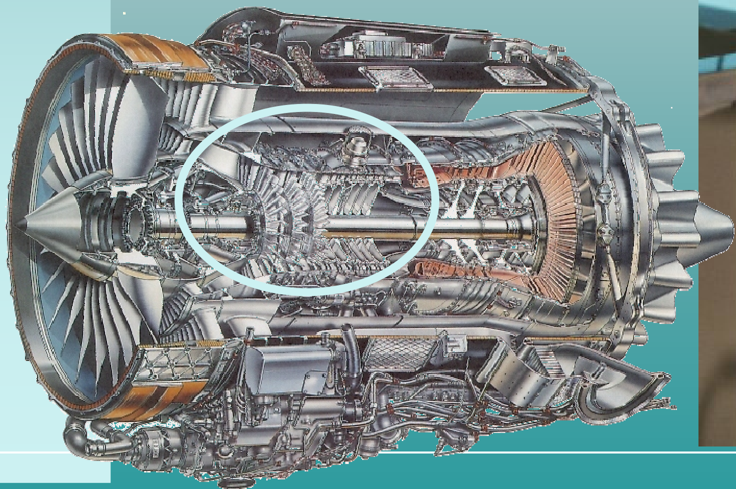
What is Optimization Good For?

- **Designing Birthday Presents**
- **Solving Problems of Existing Machines**
- **System Identification**
  - system parameters
  - dynamic behavior of passive components
  - actuator behavior
    (hydraulic, pneumatic, electro-mechanical)
  - control behavior

What is Optimization Good For?

- **Designing Birthday Presents**
- **Solving Problems of Existing Machines**
- **System Identification**
- **Virtual Prototyping**
  - multibody systems (vibrations)
  - mechatronics (control design)
  - hardware-in-the-loop optimization
  - multi-disciplinary optimization



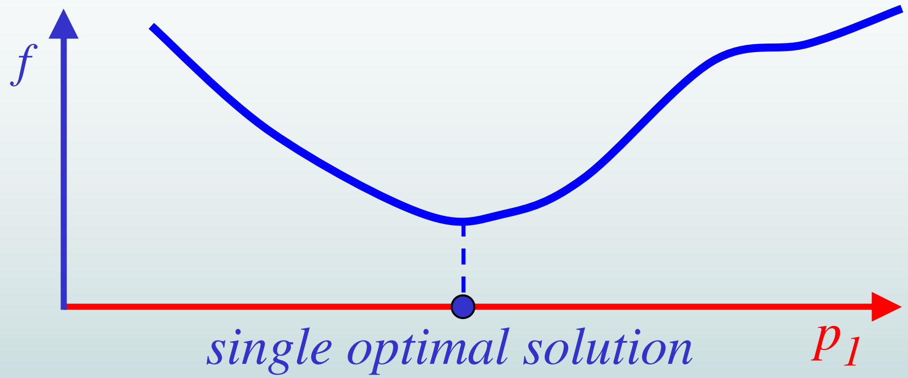


Why Multi-criterion Optimization?

a technical point of view

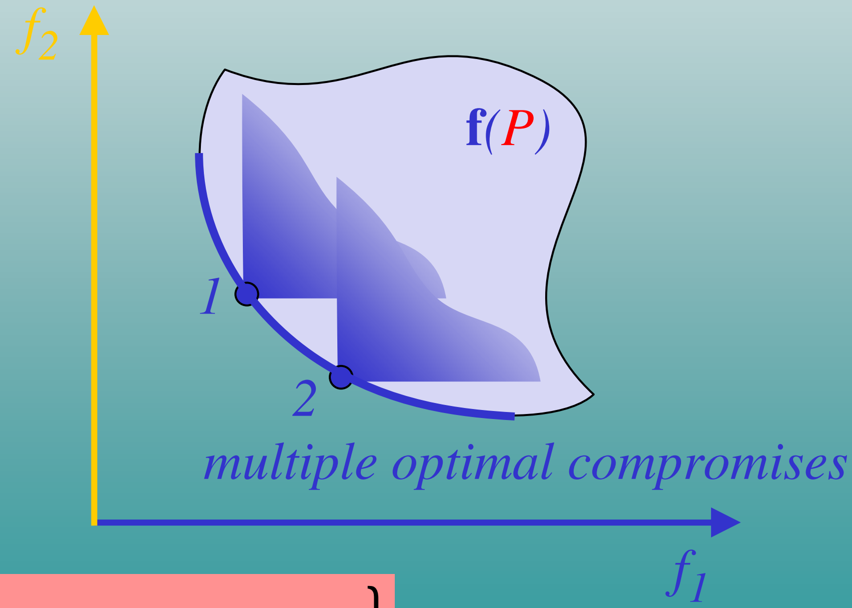
scalar optimization

$$\min_{\mathbf{p}\in P} f(\mathbf{p})$$



vector optimization

$$\min_{\mathbf{p}\in P} \underbrace{\begin{bmatrix} f_1(\mathbf{p}) \\ \vdots \\ f_n(\mathbf{p}) \end{bmatrix}}_{\mathbf{f}(\mathbf{p})}$$



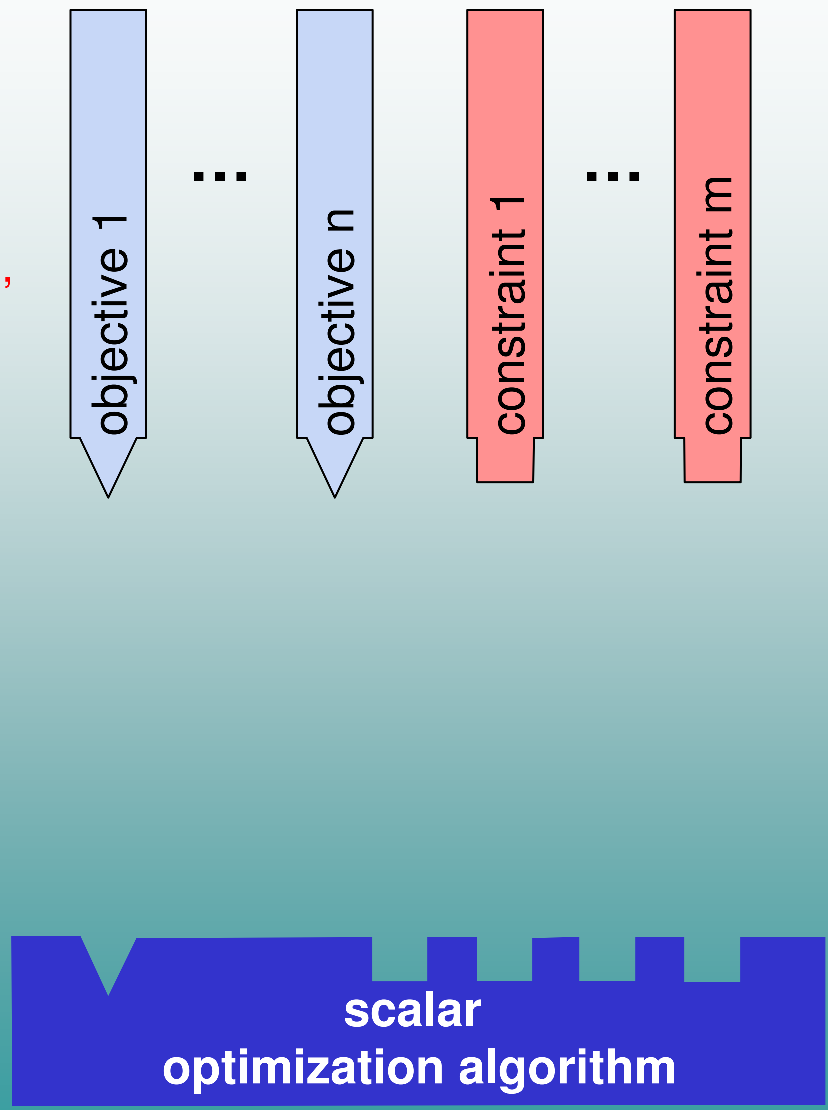
$$P = \left\{ \mathbf{p} \in I\!R^h \,\middle|\, \mathbf{g}(\mathbf{p}) = \mathbf{0},\ \mathbf{h}(\mathbf{p}) \le \mathbf{0},\ \mathbf{p}^u \le \mathbf{p} \le \mathbf{p}^o \right\}$$

Reduction Principles for Vector Optimization

vector optimization problem

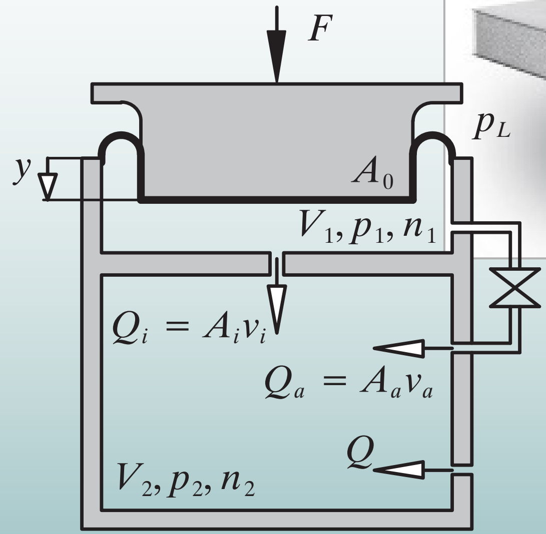
- **scalarization**
  (weighted obj., distance method, goal attainment)
- **hierarchization**
  (hierarchical opt., compromise method)
- **combination**
  (goal programming)

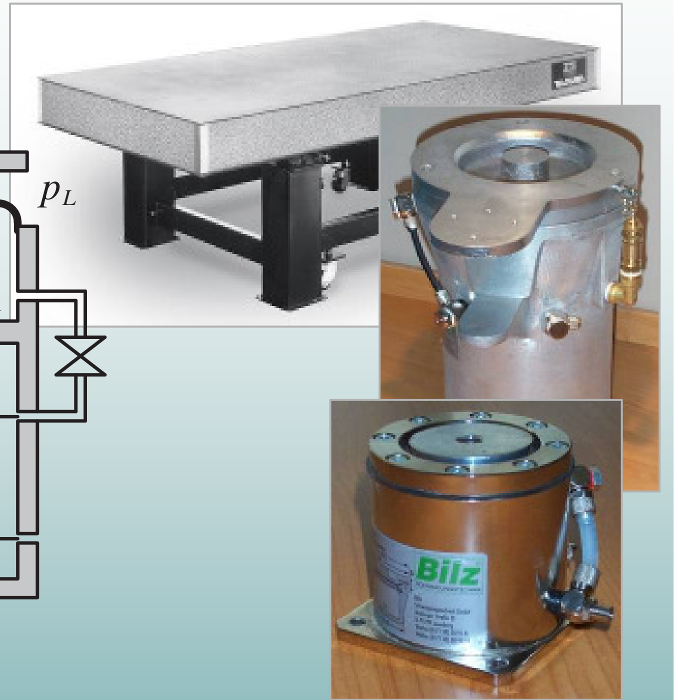
scalar optimization problem



Example 1: Horizontal Platform Insulation

problem: increase of horizontal damping





physical modeling

$$F = (p_1 - p_L) A_0$$

$$\dot{p}_1 = -\frac{n_1 p_1}{V_1} \dot{V}_1, \quad \dot{p}_2 = -\frac{n_2 p_2}{V_2} \dot{V}_2$$

$$\dot{V}_1 = -A_0 \dot{y} + q_L (p_1 - p_2)$$

$$\dot{V}_2 = -Q - q_L (p_1 - p_2)$$

Linearization

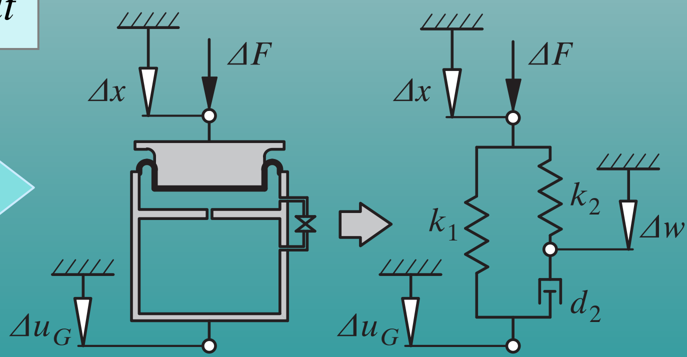
$$\Delta F = \Delta p_1 A_0$$

$$\Delta \dot{p}_1 = -q_L (\alpha + \beta) \Delta p_1 + \alpha\beta A_0 q_L \,\Delta y + \alpha A_0 \,\Delta \dot{y} + \alpha\beta q_L \int Q dt$$

Equivalent Mechanical Model (Q=0)

$$\Delta \dot{F} + \frac{k_2}{d_2} \Delta F = \frac{k_1 k_2}{d_2} \Delta y + (k_1 + k_2) \Delta \dot{y}$$

$$\Delta y = \Delta x - \Delta u_G$$

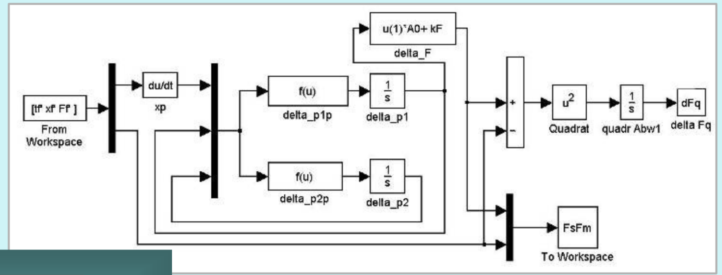


Example 1: Horizontal Platform Insulation

multi-measurement identification

experimental setup

Simulink model



F_{meas}

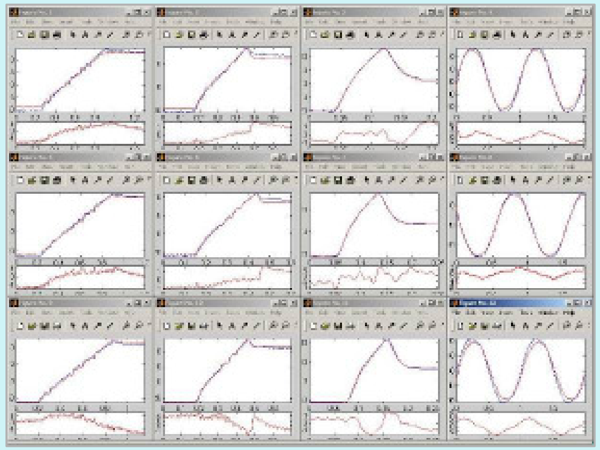
optimization (Matlab)

F_{sim}

objectives $u := \sum_i w_i \varphi_i \,,\; w_i \geq 0$

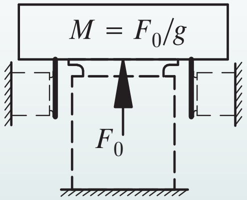
$$u := \left(\sum_i \left| \varphi_i - \varphi_{i0} \right|^r \right)^{1/r} , \; r \geq 1$$

$$where \quad \varphi_i = \sqrt{\frac{1}{T} \int_0^T \left(\frac{F_{sim,i} - F_{meas,i}}{\bar{F}_{meas,i}} \right)^2 dt}$$

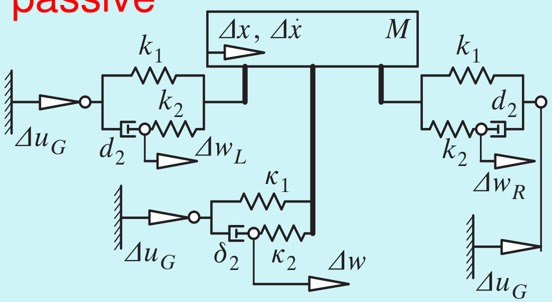


Example 1: Horizontal Platform Insulation

platform insulation



passive



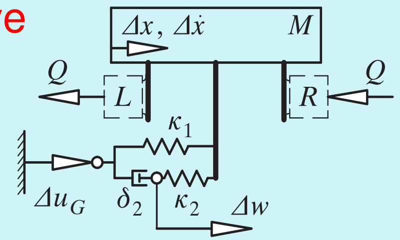
$$\mathbf{p} = \begin{bmatrix} p_0 & s_{BV} \end{bmatrix}^T$$

$$4\,\text{bar} \le p_0 \le 6\,\text{bar}$$

$$0 \le s_{BV} \le 6$$

$$\mathrm{Re}(\lambda_i) < 0$$

active



$$\mathbf{p} = \begin{bmatrix} p_0 & D \end{bmatrix}^T$$

$$4\,\text{bar} \le p_0 \le 6\,\text{bar}$$

$$-0.012 \le D \le 0.01$$

$$\mathrm{Re}(\lambda_i) < 0$$

stream flow control $Q = D\,\Delta\dot{x}$

objectives to be minimized

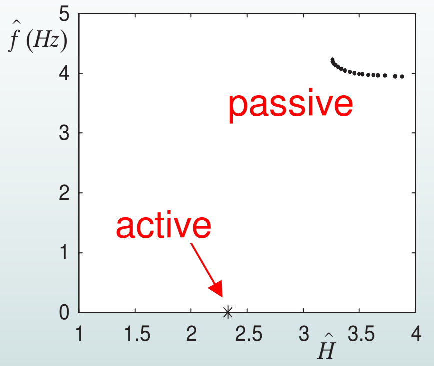
$$\hat{H} = \max_{\omega} |H(i\omega)|$$

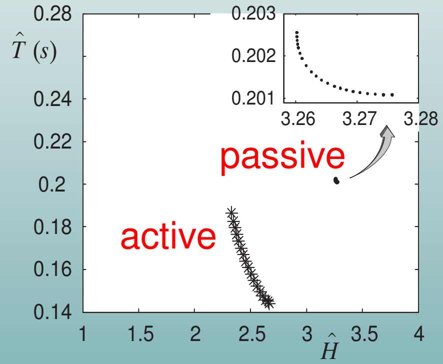
$$\hat{f} = \hat{\omega}/2\pi: \quad |H(i\hat{\omega})| = \max_{\omega} |H(i\omega)|$$

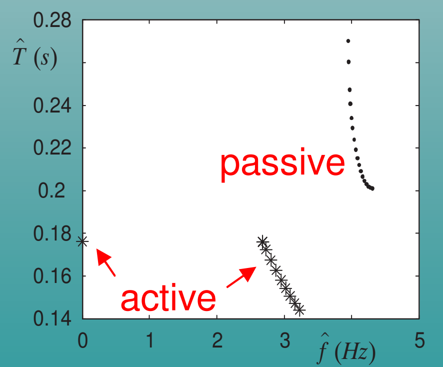
$$\hat{T} = -\frac{1}{\max_i(\mathrm{Re}(\lambda_i))}$$

bi-criterion problems

$$\min_{\mathbf{p}\in\wp} \begin{bmatrix} \hat{H} \\ \hat{f} \end{bmatrix}, \quad \min_{\mathbf{p}\in\wp} \begin{bmatrix} \hat{H} \\ \hat{T} \end{bmatrix}, \quad \min_{\mathbf{p}\in\wp} \begin{bmatrix} \hat{f} \\ \hat{T} \end{bmatrix}$$







Normal Boundary Intersection Approach

related to I. Das and J.E. Dennis

single EP-solution

$$\min_{\mathbf{p}\in P,\, t} \; t$$

$$s.t. \quad \mathbf{F}\boldsymbol{\beta} + t\,\mathbf{n} = \mathbf{f}(\mathbf{p})$$

$$\boldsymbol{\beta} = const.$$

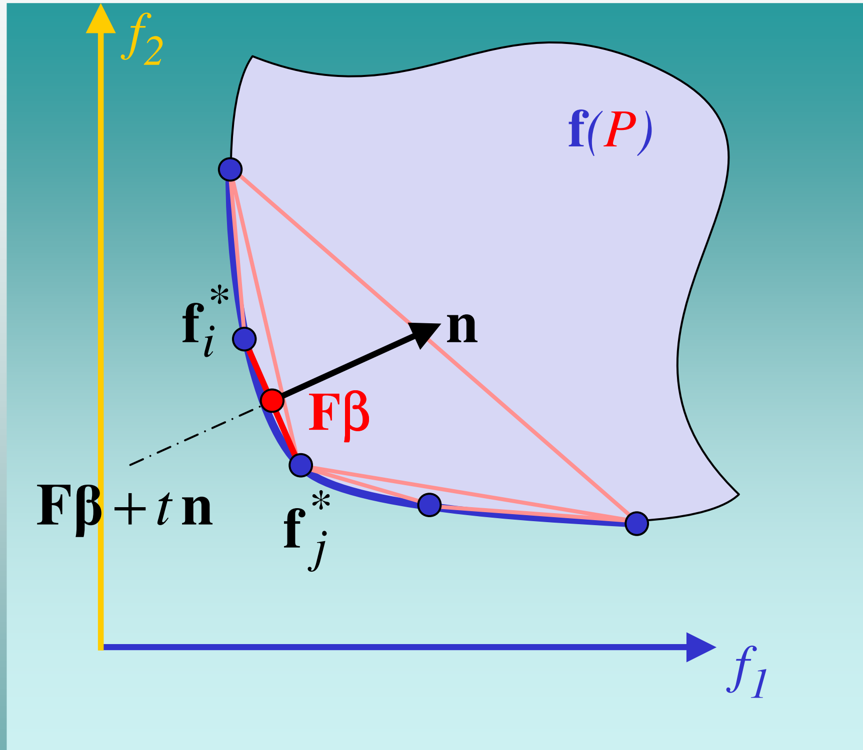
knee search

$$\min_{\mathbf{p}\in P,\, t,\, \boldsymbol{\beta}} \; t$$

$$s.t. \quad \mathbf{F}\boldsymbol{\beta} + t\,\mathbf{n} = \mathbf{f}(\mathbf{p})$$

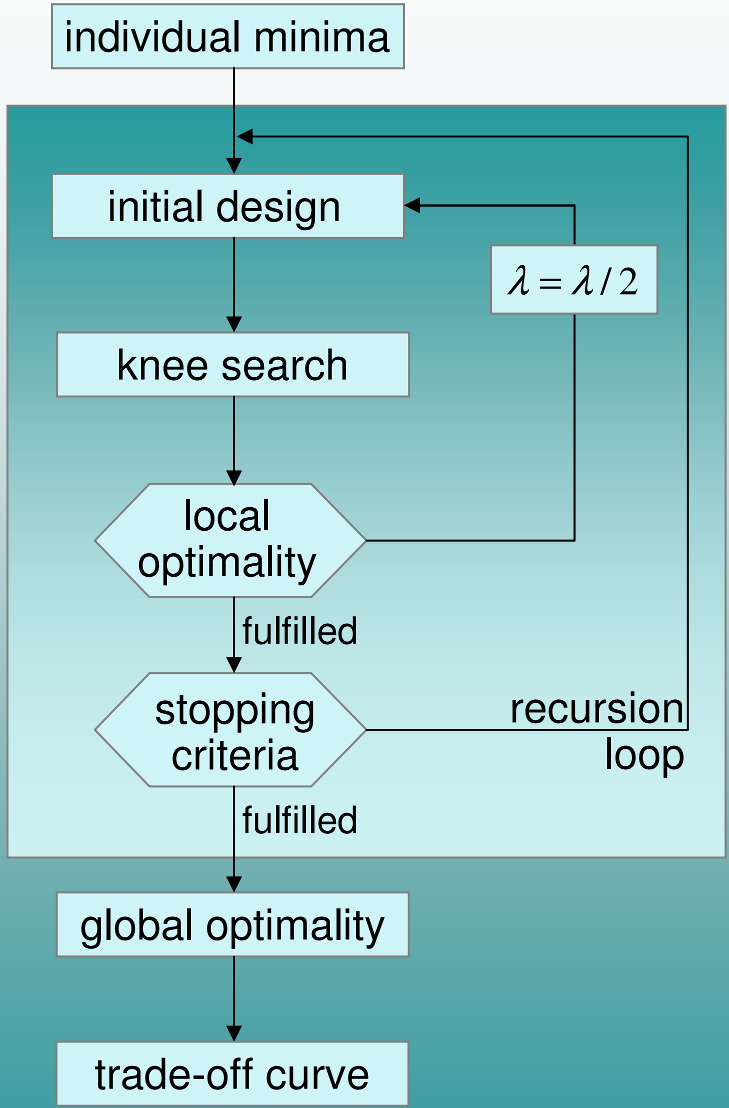
$$\beta_i \geq 0, \; \sum \beta_i = 1$$

recursive knee search



$$\mathbf{F} := \left[\; \mathbf{f}_i^* \;\; \mathbf{f}_j^* \;\right]$$

Recursive Knee Search



- initial design

$$\mathbf{p}^{(0)} = \mathbf{p}_i^* + \lambda\left[1 + \varepsilon \; rand\,(-1,1)\right]\left(\mathbf{p}_j^* - \mathbf{p}_i^*\right)$$

$$\lambda \in (0,1), \quad \varepsilon \in \left(0, \;\; \min\left\{1, \; \frac{1}{\lambda} - 1\right\}\right)$$

$$e.g. \quad \varepsilon = 0.2, \quad \lambda = 0.5$$

- bounds on knee search
  1) $t \in [-1,1]$,
  2) $\beta \in [\beta_{\min}, \; \beta_{\max}]$, $\quad e.g. \;\; \beta \in [0.4, 0.6]$

- local optimality
  1) solution feasible
  2) local EP-optimality
  3) limits on No. of non-successful restarts

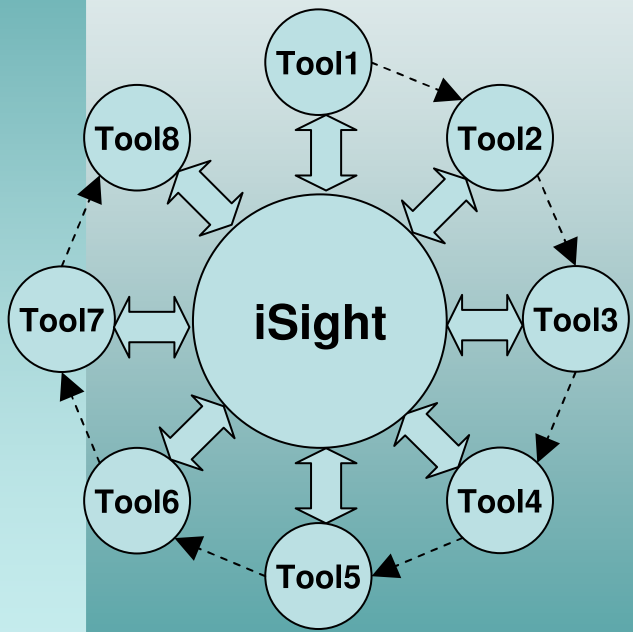
- stopping criteria
  1) $|t| \leq \bar{t}$, $\quad e.g. \;\; \bar{t} = 0.1$
  2) lower and upper recursion limits

Matlab applet

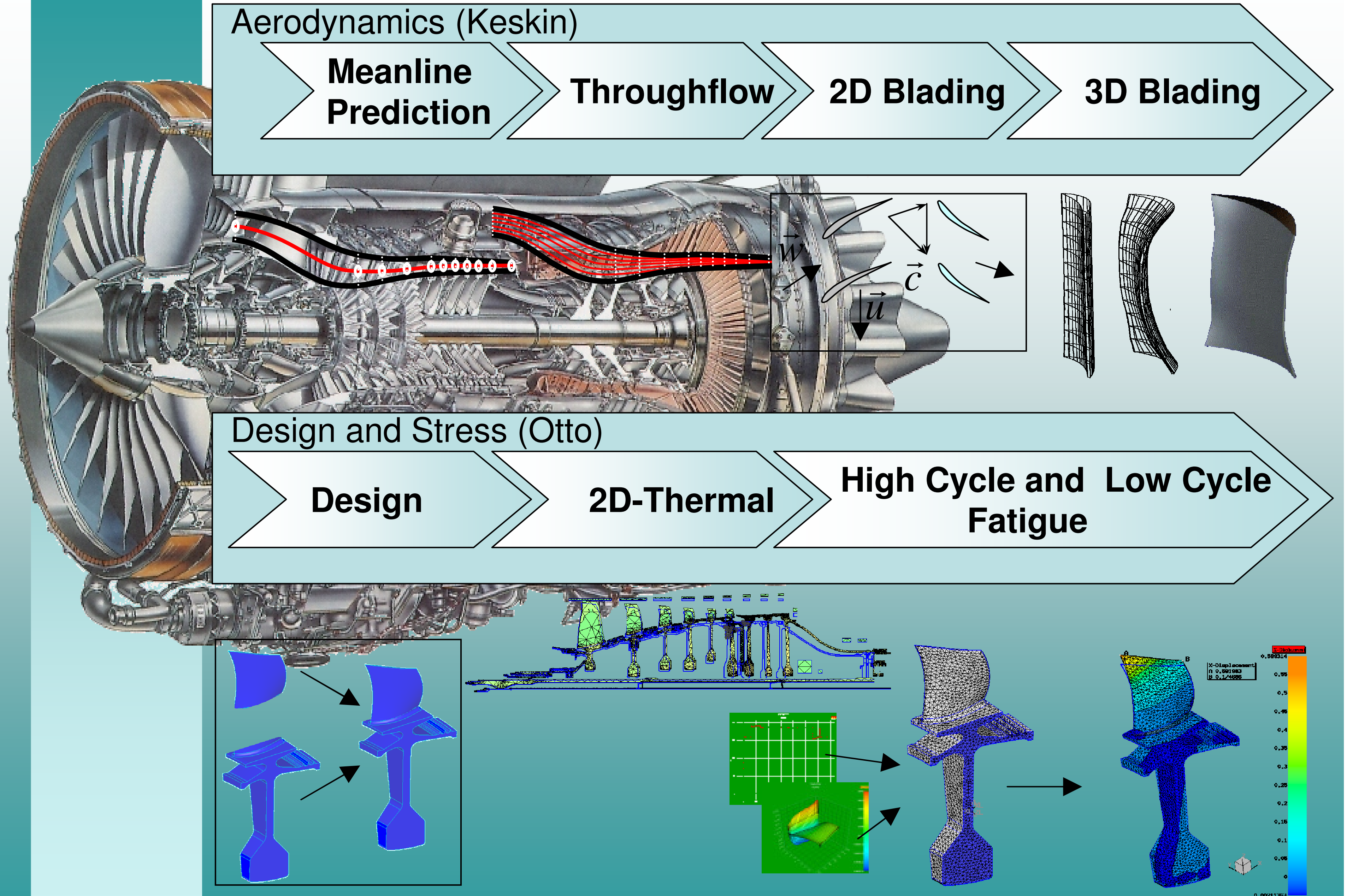
Example 2: Compressor Design Process

VIT-project (Virtual Turbomachinery, LuFo III)

Improvement of current Rolls-Royce compressor design process by tool integration and optimization



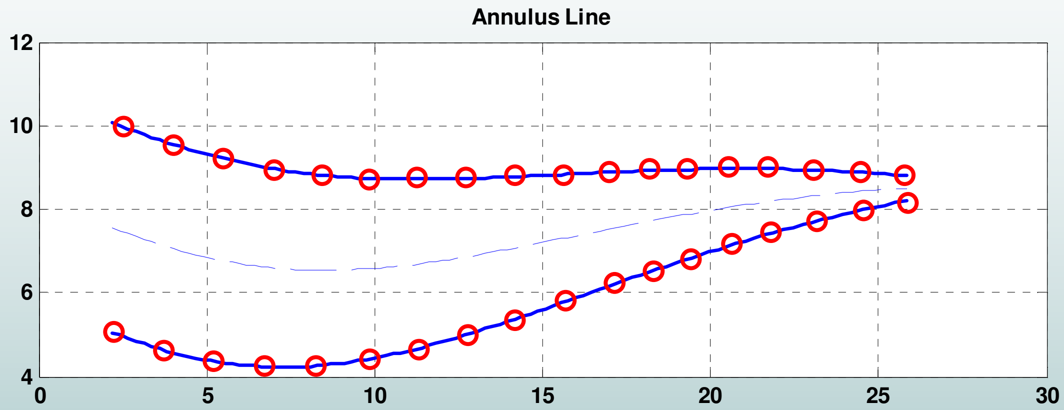
- step 1: tool by tool analysis of current compressor design process
- step 2: automation and integration of single analysis tools into an optimization environment
- step 3: partial automation of multidisciplinary design process
- the future: automation of whole multidisciplinary design process


Aerodynamics (Keskin)
Meanline Prediction
Throughflow
2D Blading
3D Blading
w
c
u
Design and Stress (Otto)
Design
2D-Thermal
High Cycle and Low Cycle Fatigue

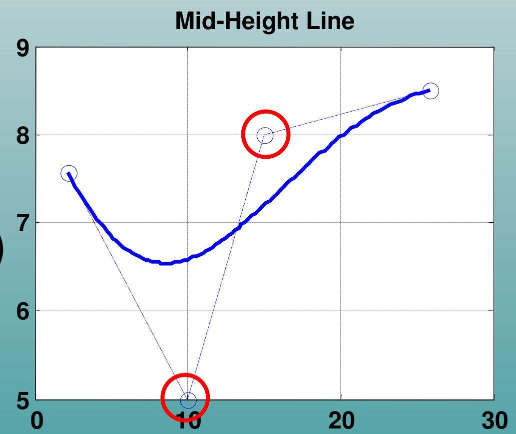
Meanline Prediction

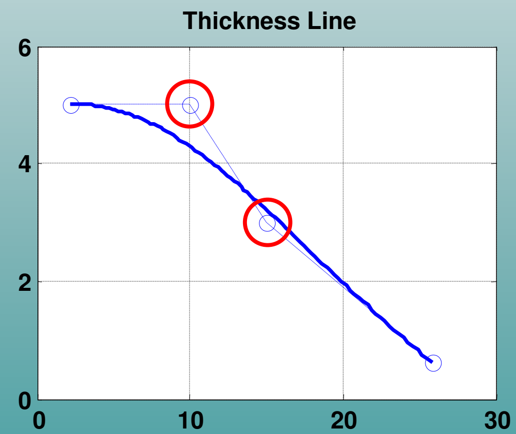
annulus parameterization

- classical point-wise definition



- Reduced parameter definition (Bézier-splines with 4 control points)





Matlab applet

$$\textit{design parameters}\ \ \mathbf{p} = \left[b_{x_2}, b_{y_2}, b_{x_3}, b_{y_3}, t_{x_2}, t_{y_2}, t_{x_3}, t_{y_3} \right]^T$$

Meanline Prediction

criteria

$$\max_{\mathbf{p}} \; \eta_{c,poly}$$

~~$\max_{\mathbf{p}} \; SM$~~ $\quad SM \geq 25\%$

constraints

$$\max_i \; \Psi_i \leq 0.6 \qquad \max_i \; DF_i^R \leq 0.55$$

$$\max_i \; M'^R_{I,i} \leq 1.1 \qquad \max_i \; DF_i^S \leq 0.55$$

$$\max_i \; M^S_{I,i} \leq 0.8 \qquad \max_i \; DH_i^R \geq 0.58$$

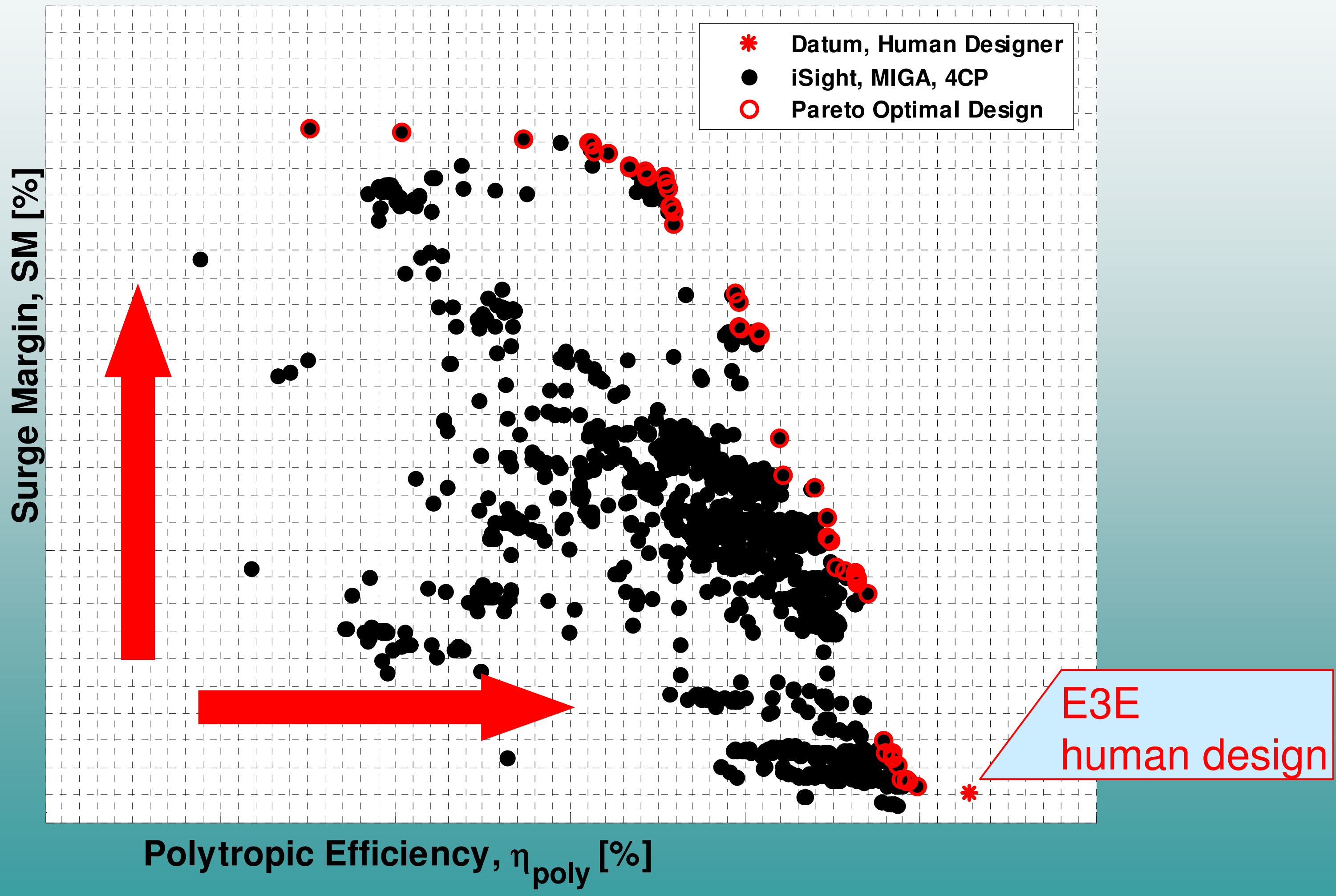
$$\max_i \; C_{h,i} \leq 0.92 \qquad \max_i \; DH_i^S \geq 0.58$$

$$M^S_{E,N_s} \leq 0.27$$

$$i \in \{1,\ldots,N_s\}$$

Meanline Prediction

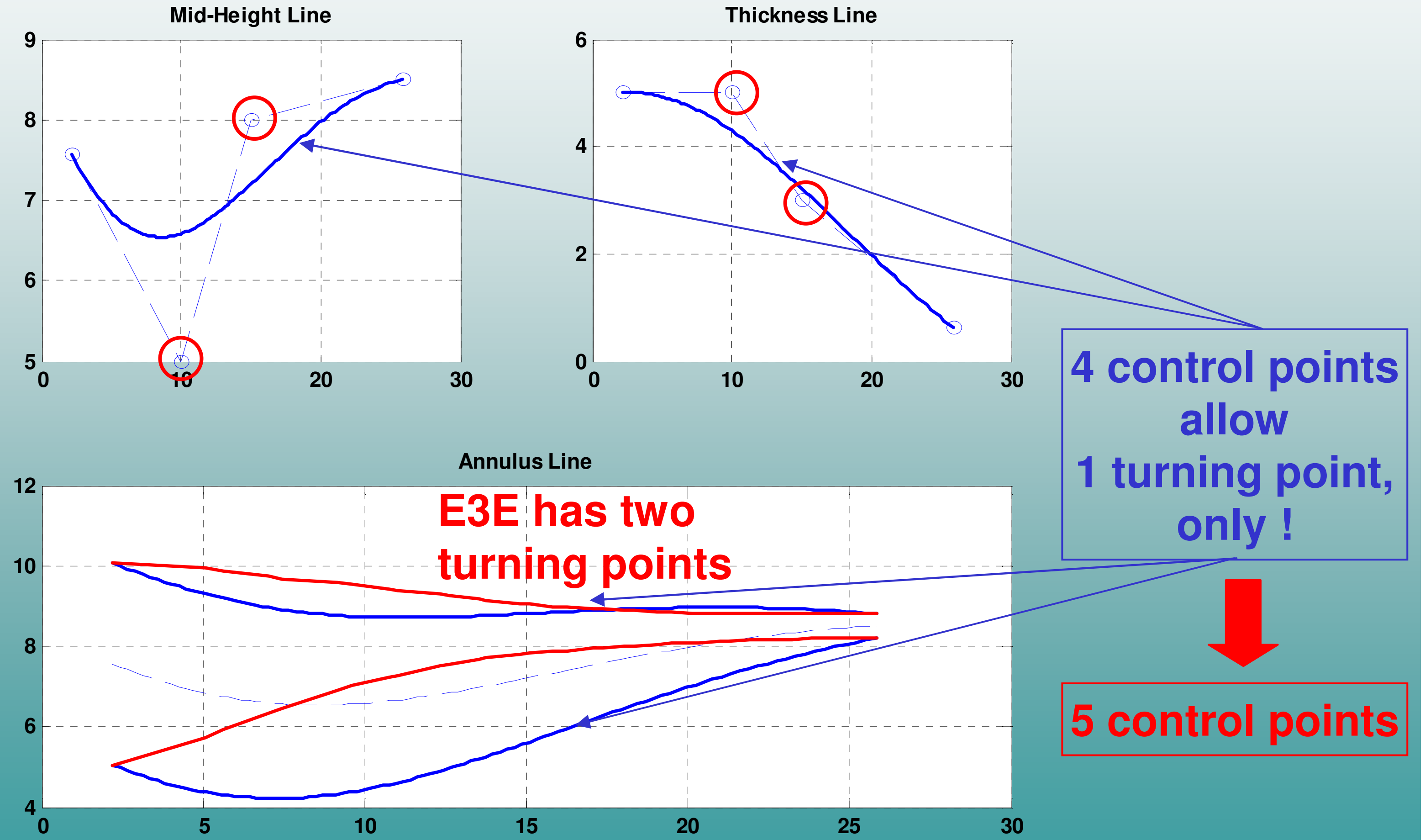
Results for Bézier-splines with 4 control points



Meanline Prediction

explanation for sub-optimal solution

1. parameterization too restrictive



Meanline Prediction

explanation for sub-optimal solution — 2. constraints too restrictive

criteria

$$\max_{\mathbf{p}} \ \eta_{c,poly}$$

~~$\max_{\mathbf{p}} \ SM$~~ $\quad SM \geq 25\%$

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$$\max_i \ \Psi_i \leq 0.6 \qquad \max_i \ DF_i^R \leq 0.55$$

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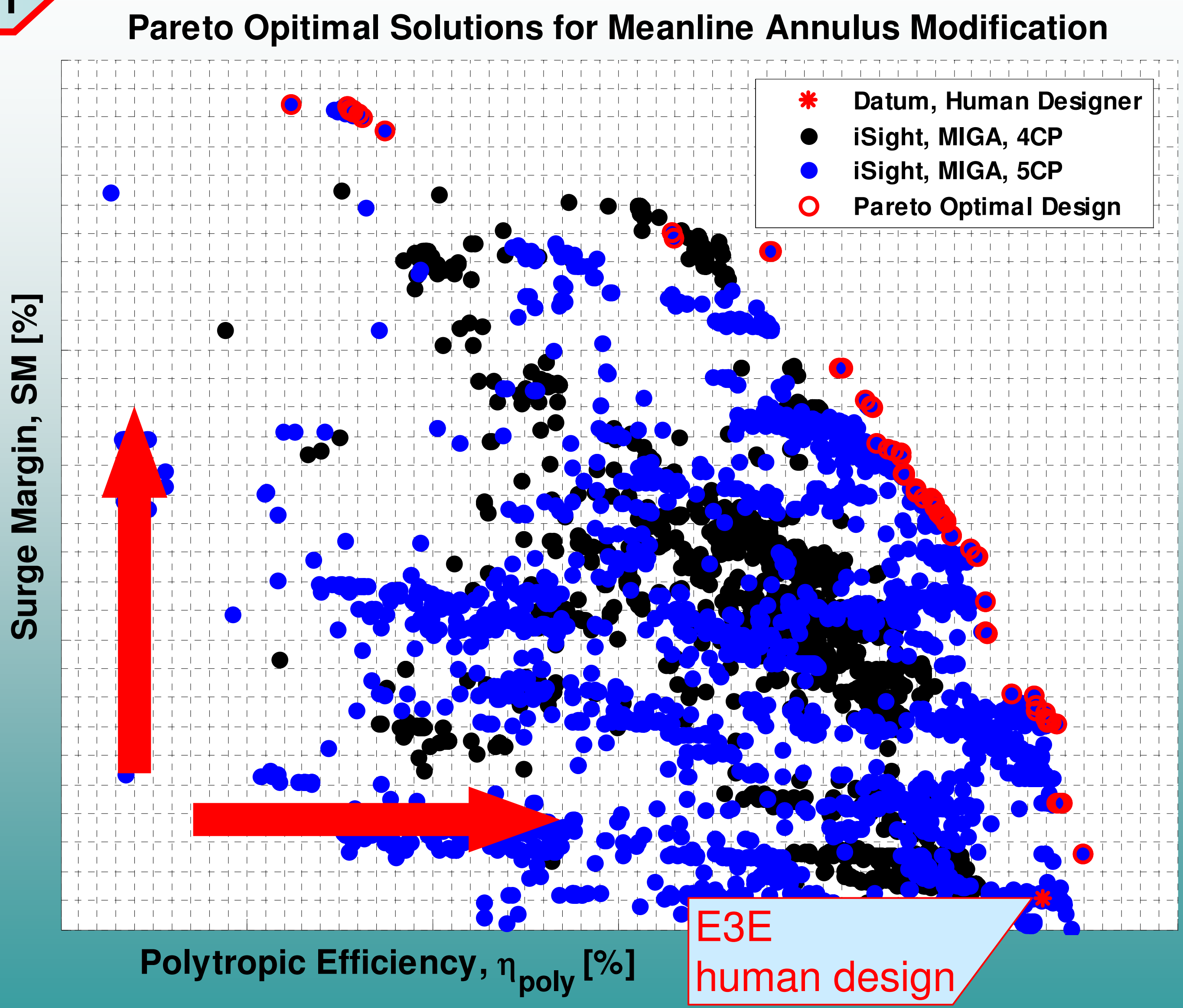
$$\max_i \ M^S_{I,i} \leq 0.8 \qquad \max_i \ DH_i^R \geq 0.58$$

$$\max_i \ C_{h,i} \leq \sout{0.92}\ 0.93 \qquad \max_i \ DH_i^S \geq 0.58$$

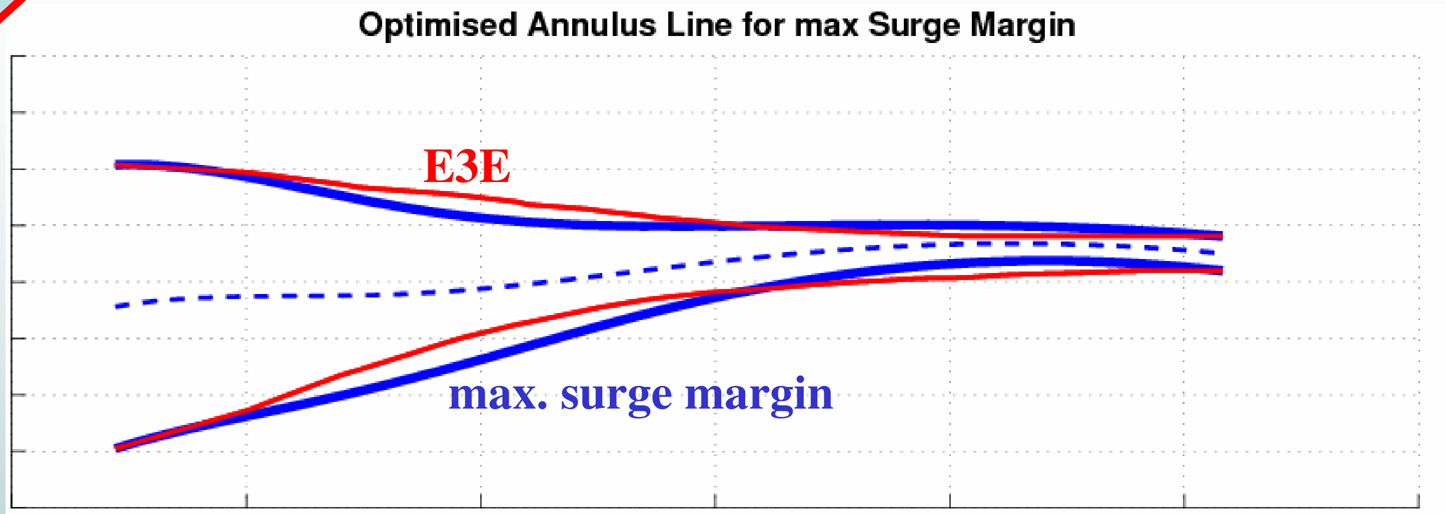
$$M^S_{E,N_s} \leq \sout{0.27}\ 0.285$$

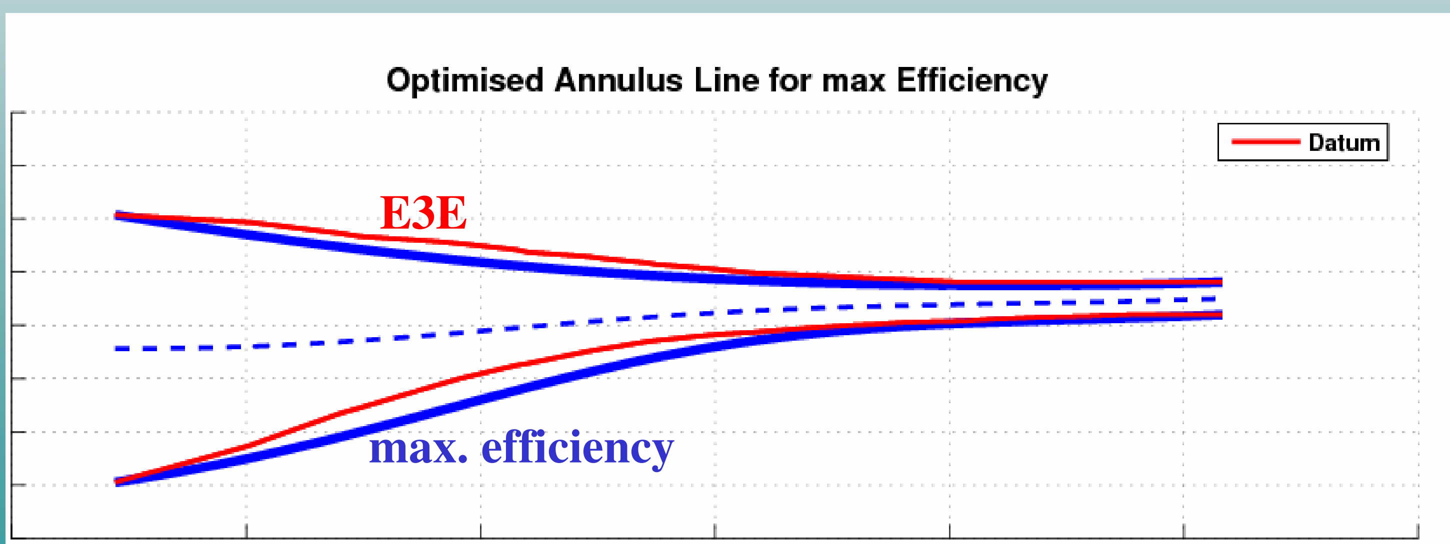
$$i \in \{1,\ldots,N_s\}$$

Meanline Prediction

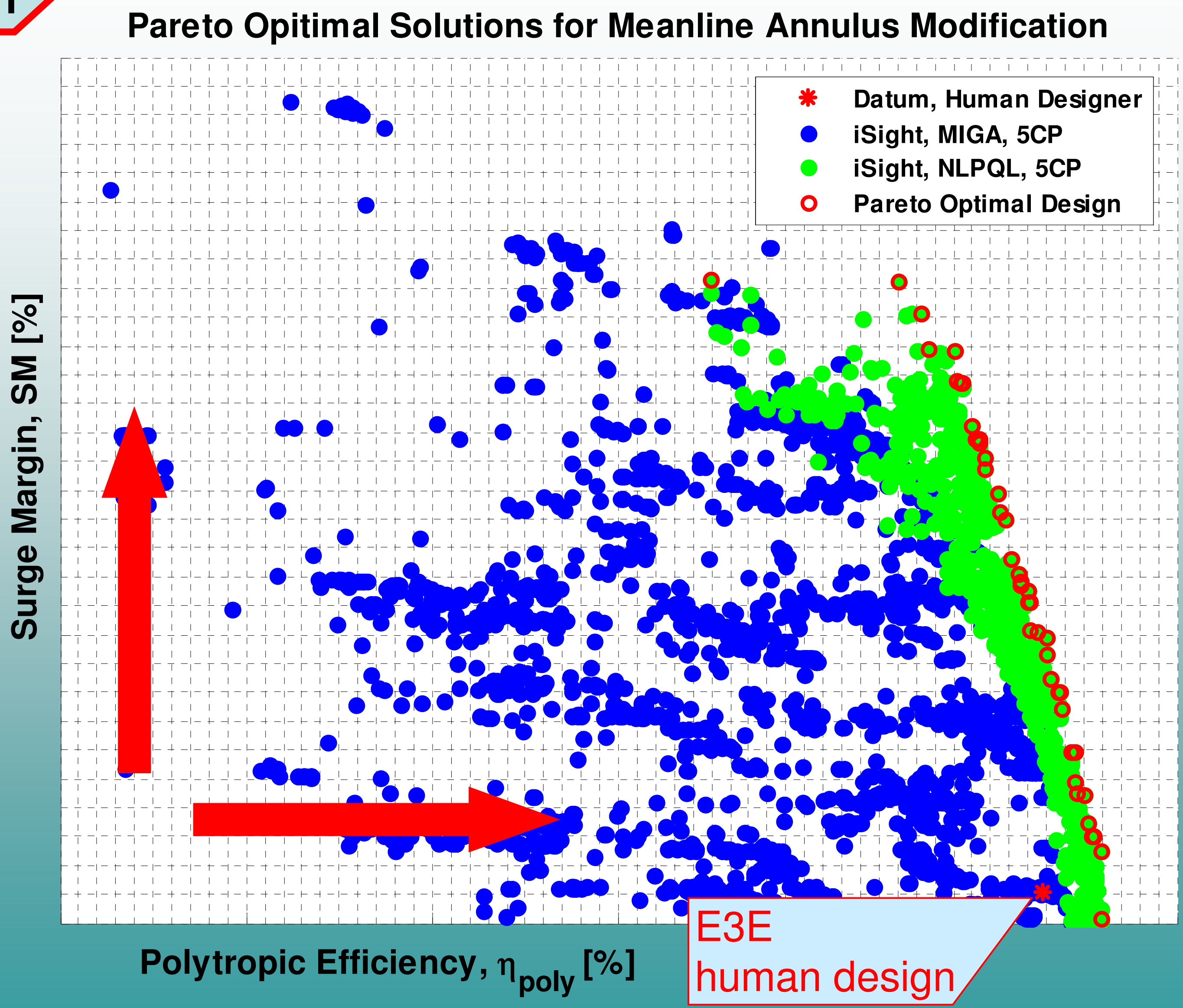


Meanline Prediction

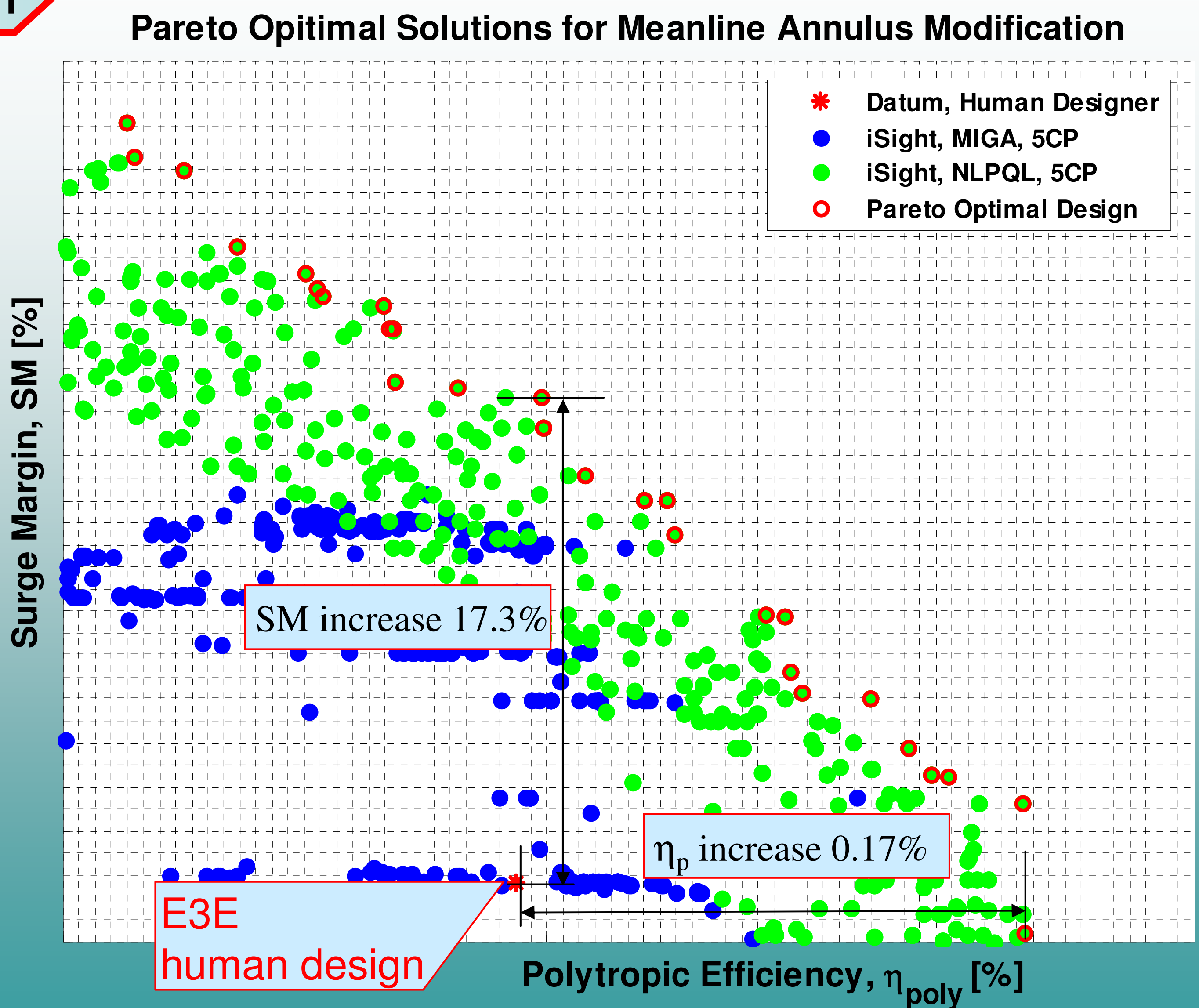




Meanline Prediction



Meanline Prediction

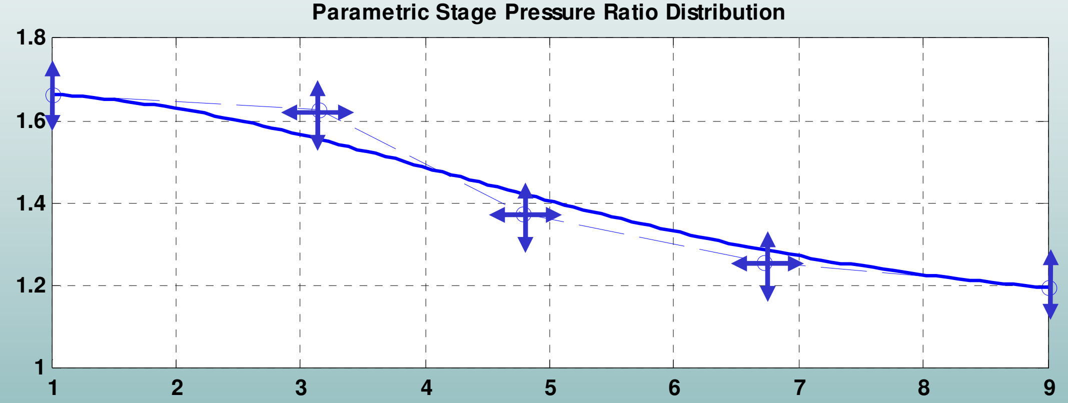


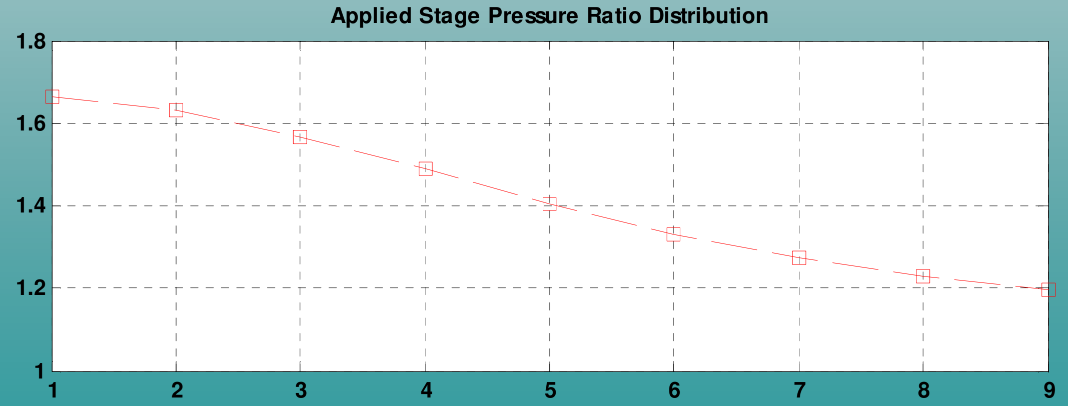
Meanline Prediction

Annulus Line & Pressure Ratio Optimization

$$\mathbf{p} = \left[b_{x_2}, b_{y_2}, b_{x_3}, b_{y_3}, b_{x_4}, b_{y_4}, t_{x_2}, t_{y_2}, t_{x_3}, t_{y_3}, t_{x_4}, t_{y_4}, p_{y_1}, p_{x_2}, p_{y_2}, p_{x_3}, p_{y_3}, p_{x_4}, p_{y_4}, p_{y_5} \right]^T$$

pressure ratio





Meanline Prediction

Annulus Line & Pressure Ratio Optimization

criteria

$$\max_{\mathbf{p}} \ \eta_{c,poly}$$

~~$\max_{\mathbf{p}} \ SM$~~ $SM \geq 25\%$

~~$\max_{\mathbf{p}} \ \Pi_c$~~ ignored

constraints

$$\max_i \ \Psi_i \leq 0.6 \qquad \max_i \ DF_i^R \leq 0.55$$

$$\max_i \ M_{I,i}^{'R} \leq 1.1 \qquad \max_i \ DF_i^S \leq 0.55$$

$$\max_i \ M_{I,i}^S \leq 0.8 \qquad \max_i \ DH_i^R \geq 0.58$$

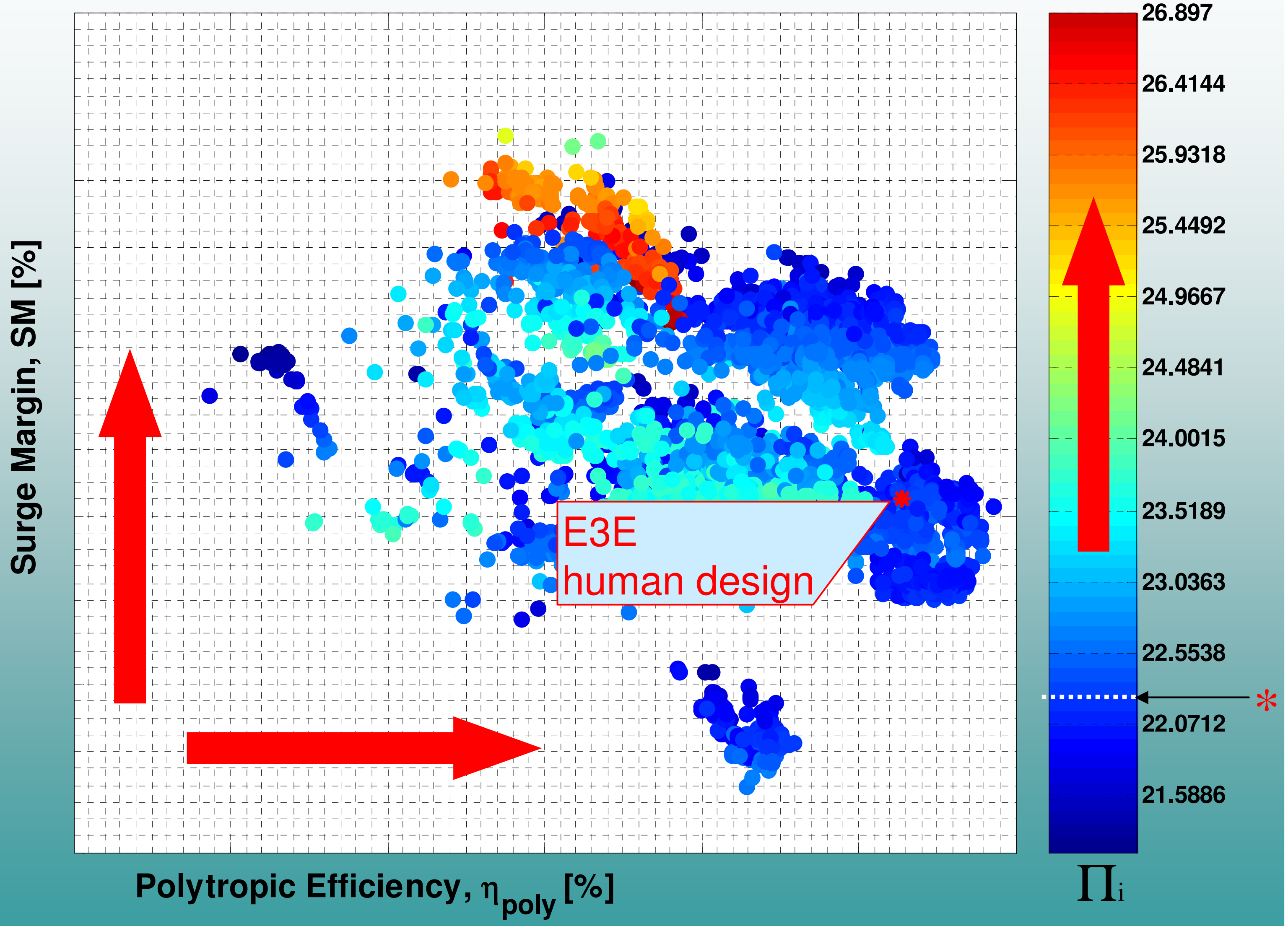
$$\max_i \ C_{h,i} \leq 0.93 \qquad \max_i \ DH_i^S \geq 0.58$$

$$M_{E,N_s}^S \leq 0.285 \qquad 1.0 \leq p_{y_j} \leq 2.0$$

$$i \in \{1,\ldots,N_s\} \quad j \in \{1,\ldots,5\}$$

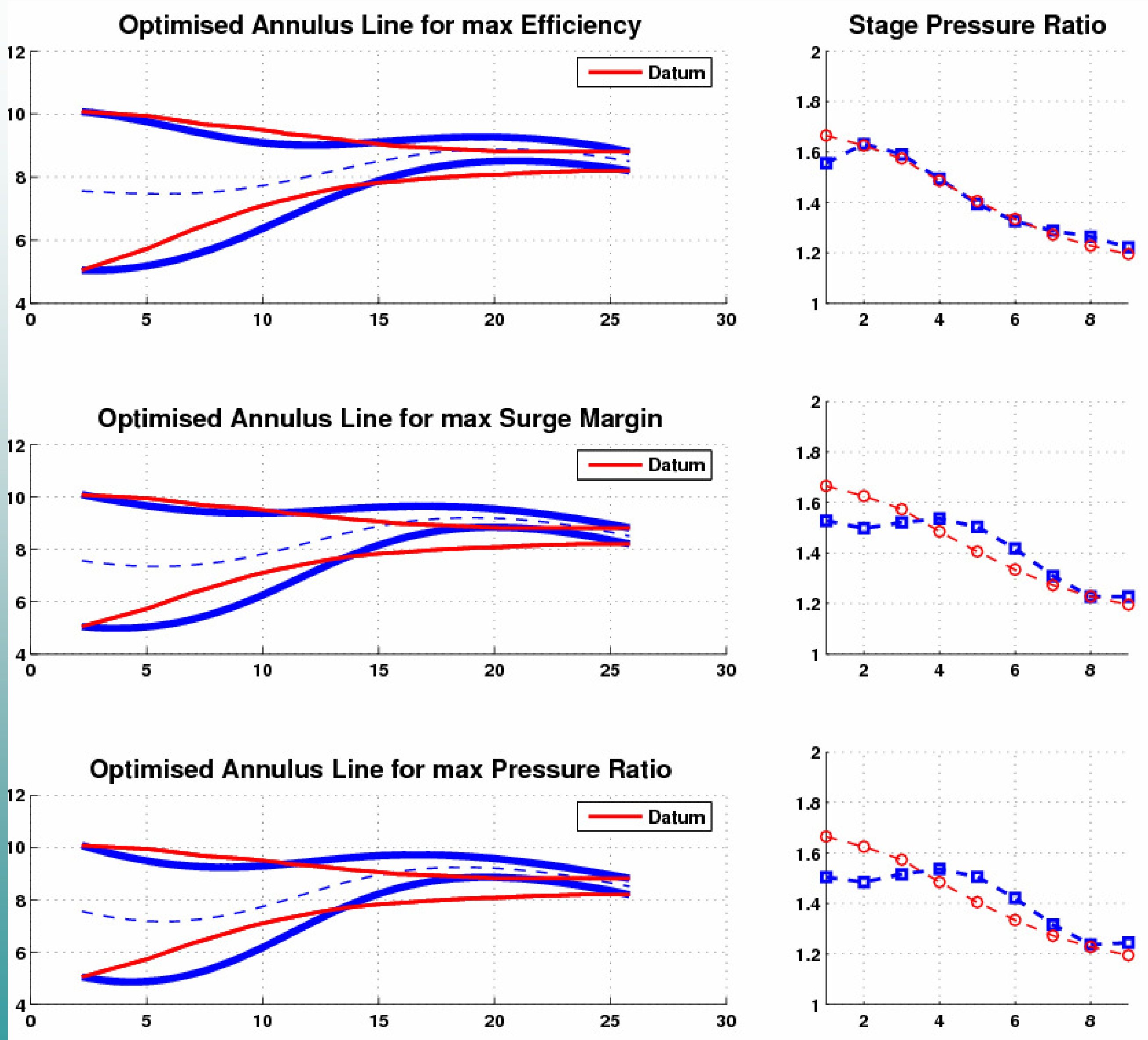
Meanline
Prediction

Annulus Line & Pressure Ratio Optimization



Meanline Prediction

Annulus Line & Pressure Ratio Optimization



Meanline Prediction

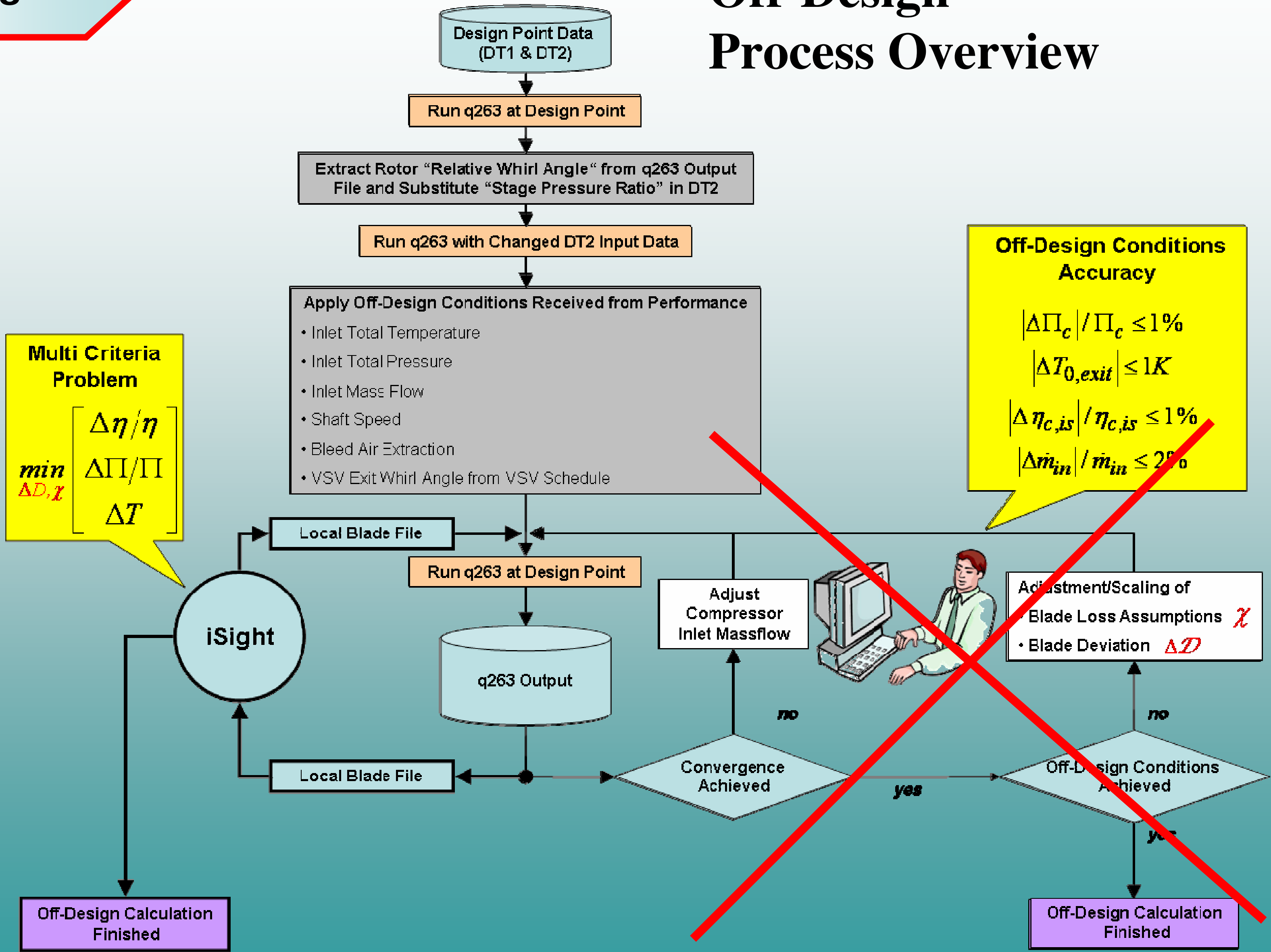
Comparison: some numbers

	function evals	converged	feasible	total time
MIGA 4CP (Annulus)	8000	7166 (~90%)	4174 (~58%)	19.3h
MIGA 5CP (Annulus)	16000	13743 (~86%)	5559 (~40%)	30.5h
NLPQL 5CP (Annulus)	10768	10637 (~99%)	839 (~8%)	18.2h
MIGA 5CP (Annulus+PR)	54000	42854 (~79%)	11909 (~28%)	94.1h

NLPQL 5CP (Annulus) best efficiency	151	151 (100%)	24 (~16%)	17min

Throughflow

Off-Design Process Overview

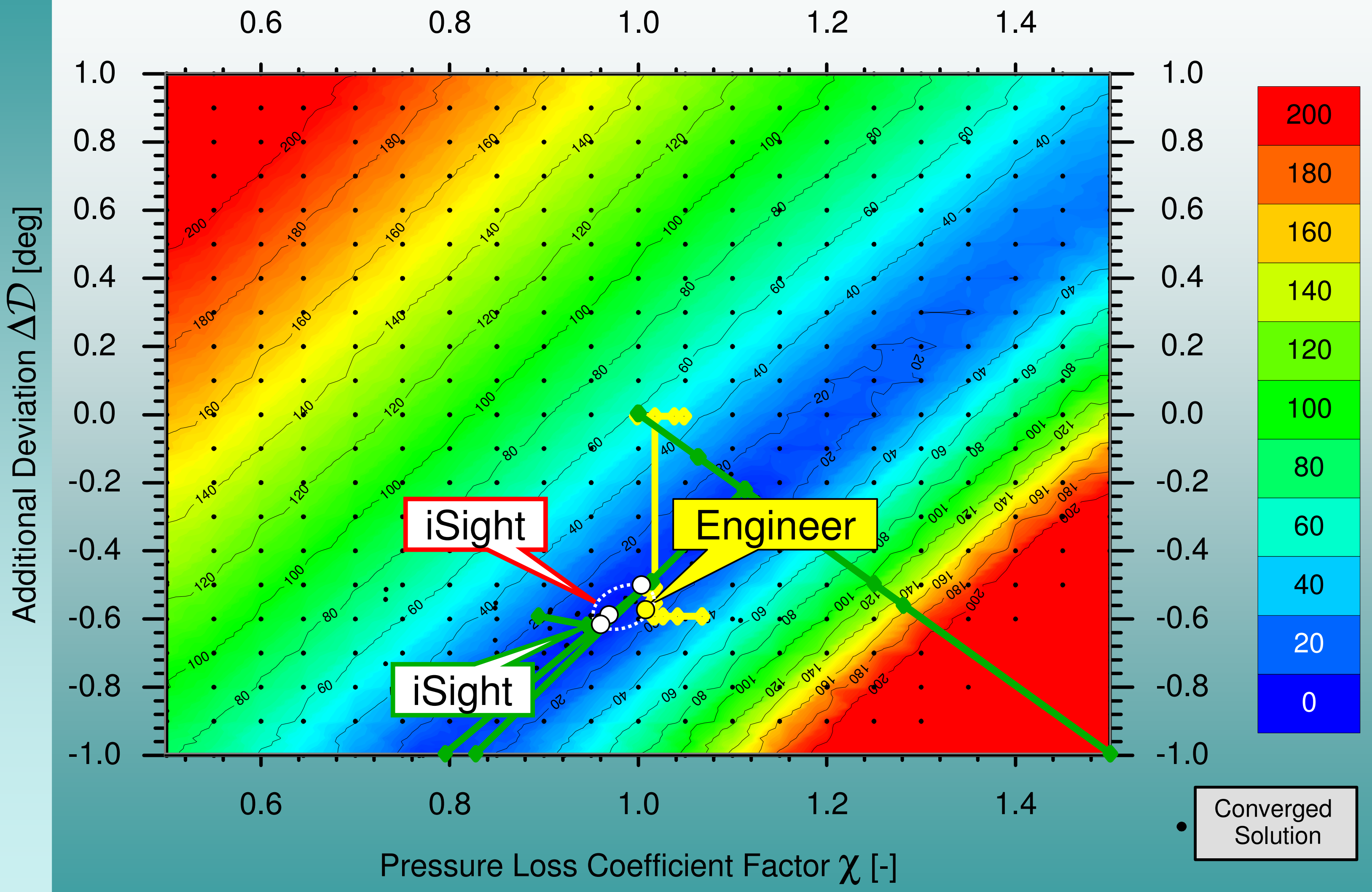


Throughflow

	human eng.	iSight (Case1)	iSight (Case2)	iSight (Case3)
No. iterations (feasible)	23	98	46	22
overall opt. time	8 h	3.6 h	1.6 h	0.76 h
time for one iteration	~21min	~2.1min	~2.1min	~2.1min
Π criterion ($\Delta\Pi/\Pi \leq 1\%$)	0.22 %	0.19 %	0.063 %	0.032 %
η criterion ($\Delta\eta/\eta \leq 1\%$)	0.53 %	0.023 %	0.544 %	0.162 %
T criterion ($\Delta T \leq 1K$)	0.83 K	0.03 K	1.2 K	0.98 K
design parameter χ	1.01857	0.964143808	1.0	0.95351
design parameter ΔD	-0.59	-0.59293051	-0.5	-0.62655
	eng.	best		fastest

multi criteria problem $\min_{\Delta D,\chi} \begin{bmatrix} \Delta\eta/\eta \\ \Delta\Pi/\Pi \\ \Delta T \end{bmatrix}$ ➡ nonlinear programming problem $\min_{\Delta D,\chi} u$ where $u = \left(100 \cdot \frac{\Delta\eta}{\eta}\right)^2 + \left(100 \cdot \frac{\Delta\Pi}{\Pi}\right)^2 + (\Delta T)^2$

Throughflow



Conclusions

- by nature, technical design problems are multi-criterion optimization problems
- multi-criterion optimization is a valuable tool for a variety of practical applications as
  - optimal system design (active and passive)
  - identification
  - search for admissible solutions (stability)
- multi-criterion optimization cuts down costs by releasing human design engineer from time-consuming parameter studies without taking him off the decision process
- multi-criterion optimization finds better results than human designer
- integrated system design allows heterogeneous analysis tools on heterogeneous platforms