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Title : Operator-splitting and ADI methods: theory and applications in multi-physics and multi-scaling problems

Abstract

In the last fifty years enormous changes have taken place in decomposition algorithms for partial differential equations. While splitting methods are attractive for decomposition into simpler equations, there has been a great deal of interest in operator-splitting methods and alternating direction methods is up to now attractive.

We collect classical and modern methods related to operator-splitting. Our contributors include both experienced researchers and young researchers adapting operator-splitting methods to new problems, e.g. nano-technology, material research, plasma-technology and others.

Since the work of Marchuk and Strang in the early sixties a there has been no attempt to publish the basic ideas in a pedagogical and research manner.

We include both the analysis of the methods and their application. So mathematicians, physicists and engineers will find the theoretical and practical background to start working with operator-splitting methods.

In this proposal we describe the analysis of numerical methods for evolution equations pedagogically using temporal and spatial decomposition methods.

Decomposition methods are discussed with respect to their effectiveness, the possibility of combining them with discretization methods, multi-scaling possibilities, stability to initial and boundary value problems, and model reducibility.

We discuss the treatment of singular, spatially-dependent, stiff and nonlinear operators because such behavior of the equations can be treated with our proposed splitting methods and their applications to multi-physics problems.

Our aim is to obtain simpler differential equations and to compute with

higher-order discretization and solver methods in order to reduce the computation and memory resources required.

The following contributions are discussed in the proposal

- Pedagogical description of each decomposition method with respect to a general overview of the related numerical analysis.
- New theoretical results of each discretization and decomposition method used.
- Discussions of efficient decomposition and discretization methods with respect to computer time and memory use.
- Presentation of embedded higher-order methods.
- Modern applications of each of splitting methods discussed, e.g. flow problems, elastic wave propagation, heat transfer, magnetic problems.

The work presented in the contributed chapters is related to real-life problems. We discuss the underlying decomposition and discretization, the stability and consistency analysis of the proposed decomposition methods and their numerical results.

In each contribution, we present the modeling of selected real-life problems. We describe the underlying characteristics of the various equation parts and their spatio-temporal behavior. This knowledge then allows us to design special decoupling methods and to respect the underlying conservation laws of physics or chemistry.

We describe the discretization and decomposition methods used to obtain decoupled equations for the contributed temporal and spatial splitting methods.

The advantages of decomposition methods include the possibility of adapting the best discretization and solver methods for the decomposed problems. The order of large problems is reduced to simpler and efficiently solvable partial problems either in space or in time.

Each contributor discusses discretization methods, e.g. finite volume and discontinuous Galerkin methods, which can be applied to real-life problems to conserve their characteristics. Further, we take into account that the decomposition methods have their advantages in decoupling the appropriate operators, with respect to their physical and mathematical behavior. Each operator can be considered with respect to his time- and spatial-scales and is discussed in the papers.

Our main contribution here is a discussion of decomposition methods. We explain the advantages of these methods over classical time-splitting methods such as operator splitting or ADI methods. An improvement of such methods to higher methods is given in the form of an intensive analysis of the iterative operator-splitting method.

A closed error analysis of the splitting methods contributed here is presented for each underlying evolution equation.

In the contributions of the splitting methods arise in two main applications:

1. Geometric integration, i.e. integration of a vector field. Applications: Hamiltonian systems, dynamical systems, symplectic splitting.

2. Decomposition methods, i.e. Operator splitting methods. Applications: Fluid dynamics, computational mechanics.

3. ADI methods (higher order methods). Applications: Combustion problems, porous media problems.

In the practical contributions are the applications of the splitting methods:

1. Porous media (advection-diffusion equations)
2. Fluid dynamics (Navier-Stokes and Boltzmann equations)
3. Quantum mechanics (Schroedinger equation)
4. Hamiltonian systems (Poisson systems)
5. Elastic wave propagation (Wave equations)
6. Molecular dynamics (Boltzmann equations and drift-diffusion)
7. Model reduction techniques (e.g. Large-scale structures, weather prediction, simulation and control of chemical reactions)
8. Combustion problems (Diffusion-Reaction-Equations)