HU Berlin, November 2022

Title: Parallel spinors on Lorentzian manifolds

Parallel spinors, i.e. parallel sections of the spinor bundle, are related to many problems in mathematics and physics. All known compact Ricci-flat Riemannian manifolds have a finite covering that admits a parallel spinor. In physics they seem to be related to supersymmetry and hidden dimensions. Parallel spinors lead to special holonomy.

In the talk we will not discuss these applications. We want to present unexpected tight relations between Lorentzian metrics of dimension $n+1$ carrying a parallel (lightlike) spinor and 1-parameter families of Riemannian metrics carrying a parallel spinors on an $(n-1)$-dimensional manifold.

The decomposition of such a Lorentzian manifold into a family of Riemannian metrics was done in work by Baum, Leistner and Lischewski (2014–2017). In joint work with Klaus Kröncke and Olaf Müller (2021) we described how to assemble a 1-parameter family of Riemannian manifolds with parallel spinors to a Lorentzian one.

However, it is not fully clear to which extent these two constructions are inverse to each other. It seemed that the Baum–Leistner–Lischewki construction which depends on the choice of a spacelike hypersurface yields a wider class of metrics than the Ammann–Kröncke–Müller construction. We will show (joint work with K. Kröncke) that a least locally this discrepancy can be removed by a clever choice of this spacelike hypersurface. This leads to a partial differential equation of Hamilton-Jacobi type that we will resolve.

The subject is also related to index theory in the Lorentzian context. Let us assume that a Lorentzian manifold with closed Cauchy surface M satisfies the “dominant energy condition”, a condition that may be physically interpreted as non-negative energy density. One can use index-theoretical arguments for the Dirac-Witten operator. If this operator has a non-trivial kernel, we obtain a Lorentzian manifold with a lightlike parallel spinor. This is joint work with my PhD student Jonathan Glöckle.