Stability of Einstein Metrics

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Einstein metrics on compact manifolds are critical points of the Einstein–Hilbert functional of General Relativity under volume preserving variations. Considered as critical points Einstein metrics are always saddle points: the Hessian of the Einstein–Hilbert functional in an Einstein metric always has infinite dimensional spaces of both positive and negative directions. Conformal volume perserving variations however account for almost all positive directions of the Hessian of the Einstein–Hilbert functional; in turn Einstein metrics are called stable, if they are local maxima of the Einstein–Hilbert functional restricted to volume preserving variations transversal to their conformal class.

In my talk I want to discuss recent joint work with Uwe Semmelmann and Paul Schwahn on the stability of Einstein metrics on symmetric and homogeneous spaces. In particular we showed that the Einstein metric on the Cayley projective plane is stable, whereas the Einstein metrics on the quaternionic Grassmannians different from quaternionic projective spaces are unstable. Moreover we used properties of the standard Laplace operator to prove the stability of the Einstein metric on the homogeneous, non–symmetric space $\mathbf{E}_7/\mathbf{PSO}(8)$, making this space the first known example of a non–symmetric manifold with positive scalar curvature and stable Einstein metric.