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Generalisations of 3-Sasakian manifolds and their compatible connections with skew torsion

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3-Sasaki manifolds are a very well-studied, but unfortunately rather rigid class of almost 3-contact metric manifolds (for example, they are always Einstein, and they are indeed mathematically rigid by a result of Pedersen and Poon). In the first part of this talk, we define and investigate new classes of almost 3-contact metric manifolds, in particular 3-(α, δ)-Sasaki manifolds (including as special cases 3-α-Sasaki manifolds, quaternionic Heisenberg groups, and many others) and prove that they are hypernormal, thus generalizing a seminal result by Kashiwada. We study their behaviour under a new class of deformations, called H-homothetic deformations, and prove that they admit an underlying quaternionic contact structure, from which we deduce the Ricci curvature.

The second part is actually devoted to finding these 'good' compatible connections. We start with a very general notion of φ-compatible connections, where φ denotes any element of the associated sphere of almost contact structures. For 3-(α, δ)-Sasaki manifolds, we compute the torsion of this connection explicitly and we prove that it is parallel, we describe the holonomy, the ∇-Ricci curvature, and we show that the metric cone is a HKT-manifold. In dimension 7, we construct a cocalibrated G_2-structure inducing the canonical connection and we prove the existence of four generalized Killing spinors. This is joint work with Giulia Dileo (University of Bari, Italy).