Spectral asymptotics of Robin Laplacians on polygonal domains

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Abstract

Let $\Omega \subset \mathbb{R}^2$ be a curvilinear polygon and Q^γ_Ω be the Laplacian in $L^2(\Omega)$, $Q^\gamma_\Omega \psi = -\Delta\psi$, with the Robin boundary condition $\partial_\nu \psi = \gamma\psi$, where ∂_ν is the outer normal derivative and $\gamma > 0$. We are interested in the behavior of the eigenvalues of Q^γ_Ω as γ becomes large. We prove that there exists $N_\Omega \in \mathbb{N}$ such that the asymptotics of the N_Ω first eigenvalues of Q^γ_Ω is determined at the leading order by those of model operators associated with the vertices: the Robin Laplacians acting on the tangent sectors associated with $\partial\Omega$. In the particular case of a polygon with straight edges the N_Ω first eigenpairs are exponentially close to those of the model operators. Moreover, if the polygon admits only *non-resonant* or concave corners, we prove that, for any fixed $j \in \mathbb{N}$, the $N_\Omega + j$ eigenvalue $E_{N_\Omega+j}(Q^\gamma_\Omega)$ behaves as

$$E_{N_\Omega+j}(Q^\gamma_\Omega) = -\gamma^2 + \mu^D_j + o(1), \text{ as } \gamma \to +\infty,$$

where μ^D_j stands for the jth eigenvalue of the operator $D_1 \oplus ... \oplus D_M$ and D_n denotes the one-dimensional Laplacian $f \mapsto -f''$ on $(0, l_n)$, where l_n is the length of the nth side of Ω, with the Dirichlet boundary condition. Finally, we prove a Weyl asymptotics for the eigenvalue counting function of Q^γ_Ω for a threshold depending on γ, and show that the leading term is the same as for smooth domains.