Representation equivalence and p-spectrum of constant curvature space forms

Emilio Lauret (Universidad Nacional de Córdoba, Argentina)

Let $X = G/K$ be a symmetric Riemannian space where G is its isometry group. We shall consider discrete cocompact subgroups Γ of G acting without fixed points on X. For a finite dimensional unitary representation τ of K there are two notions that come from harmonic analysis:

- Γ_1 and Γ_2 are called τ-isospectral if the Laplace operators acting on the sections of the vector bundle E_τ of $\Gamma_i\backslash X$ associated to τ have the same spectrum;
- Γ_1 and Γ_2 are called τ-representation equivalent if $n_{\Gamma_1}(\pi) = n_{\Gamma_2}(\pi)$ for every unitary representation π of G that its restriction on K contains τ.

Here $n_\Gamma(\pi)$ denotes the multiplicity of π in the representation $L^2(\Gamma\backslash G)$.

We shall study the relations and differences between these two notions in the particular case of constant curvature space forms (i.e. $X = S^n, \mathbb{R}^n, H^n$) and $\tau = \tau_p$ the complexified p-exterior representation of $\mathrm{O}(n)$. We will first review Pesce's work. He proved that τ-representation equivalence implies τ-isospectrality and the converse is true for the trivial representation $\tau = 1_K$ in constant curvature space forms. We extend this last result in the spherical case for $\tau = \tau_p$. For nonpositive curvature space forms, we will give examples showing that τ_p-isospectrality is far from implying τ_p-equivalence, but a variant of Pesce's result remains true.

This is a joint work with Roberto Miatello and Juan Pablo Rossetti.