

A backward lambda-Lemma for the forward heat flow

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Fix a closed Riemannian manifold M and consider the classical action functional given on a loop in M by integrating kinetic minus potential energy along the loop. Here we assume that the potential V is a time dependent function on M such that the action is a Morse functional. Then the downward L^2 -gradient of the action generates an infinite dimensional hyperbolic dynamical system on the loop space of M , namely the well known (forward) heat semi-flow. Its fixed points are the critical points of the action and these are the (perturbed) closed geodesics in M .

A fundamental tool in finite dimensional dynamical systems is the lambda-Lemma which asserts that any disk transversal to the unstable manifold of a hyperbolic fixed point and of complementary dimension C^1 -converges under the backward flow locally to the stable manifold. We prove an analogue for the heat flow although there is no backward flow.

The first application is a construction of a Morse filtration of the loop space which has the property that the boundary operator of the associated cellular complex is given by counting downward heat flow lines with signs. Since the cellular chain groups are generated by the critical points of the action and graded by the Morse index we obtain a natural isomorphism between singular homology of the loop space and Morse homology associated to the downward L^2 -gradient of the action functional.