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INTRODUCTION TO SPECTRAL THEORY OF DIFFERENTIAL OPERATORS, SS 2011

Exercises 1, week of April 25, 2011

1 Finite dimensional spaces
Let $\dim E < \infty$ and denote by $E^\star$ the dual space.

a) Show that any two norms on E are equivalent.

b) Show that any finite dimensional subspace of a B-space is closed.

c) Show that any finite dimensional subspace E_1 of a B-space E admits a complement E_2, i.e. a closed subspace of E such that

$$E_1 \cap E_2 = 0, \quad E_1 \oplus E_2 = E.$$

d) Show that any closed subspace E_1 of a B-space E with finite codimension,

$$\dim E/E_1 < \infty,$$

admits a complement E_2.

2 Let M be an open subset of $\mathbb{R}^m$ with compact closure and let $k \in C(\overline{M} \times \overline{M})$. Show that the map

$$L^2(M) \ni f \mapsto Kf \in L^2(M),$$
$$Kf(X) := \int_M k(x,y)f(y)dy$$

defines a compact operator. Can this statement be extended to $k \in L^\infty(M \times M)$?

3 Let E be a Banach space. Show that $\mathcal{K}(E)$ is a twosided $\star$ ideal.

4 Show that a projection P in a Banach space E is in $\mathcal{K}(E)$ iff $\dim P(E) < \infty$.