Prof. Dr. J. Sprekels
Humboldt-Universität zu Berlin
Institut für Mathematik

"Partial Differential Equations" (BMS)
– SS 10 –
Exercises No. 1

to be presented on Tuesday 27 April, 2010

Exercise 1:

Solve the following PDEs treating them as ODEs (using integration).

a) $u_{xx}(x,y) + u_x(x,y) = 0$;

b) $u_{xy}(x,y) + yu_x(x,y) = 0$;

c) $u_{xx}(x,y) + u(x,y) = 0$.

Exercise 2:

Consider the PDE

$$\Delta u = 0 \;\text{ in }\; \Omega = \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 < 1\}, \qquad u = f \;\text{ on }\; \partial\Omega$$

with f of the form$f(\varphi)$ satisfying $f(0) = f(2\pi)$.
Show that transformation to polar coordinates yields

$$u_{rr}(r,\varphi) + \frac{1}{r}u_r(r,\varphi) + \frac{1}{r^2}u_{\varphi\varphi}(r,\varphi) = 0, \qquad u(1,\varphi) = f(\varphi)$$

for $0 < r < 1$, $0 \leq \varphi < 2\pi$.

Exercise 3:

Find solutions of the PDE

$$x^2 u_x(x,y) + \frac{1}{y}u_y(x,y) + a \cdot u(x,y) = 0, \quad a \in \mathbb{R}$$

making the ansatz $u(x,y) = X(x)Y(y)$.