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"Partial Differential Equations" (BMS)
– SS 10 –
Exercises No. 3

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Please note, that first two excises count as one for the presentation

Exercise 7:

Let $u : \mathbb{R}^2 \to \mathbb{R}$ be sufficiently smooth and $\tilde{u}(\xi, \eta) = u(x,t)$ for the coordinate transformation $\xi = x + at$, $\eta = x - at$ with $a \neq 0$.
Show that then holds:

$$u_{tt}(x,t) = a^2 u_{xx}(x,t) \qquad \text{if and only if} \qquad \tilde{u}_{\xi\eta}(\xi,\eta) = 0.$$

Exercise 8:

Let $\varphi, \psi : \mathbb{R} \to \mathbb{R}$, $x_0 \in \mathbb{R}$ fixed and, for some $a \neq 0$,

$$u(x,t) = \frac{\varphi(x+at) + \varphi(x-at)}{2} + \frac{1}{2a} \int\limits_{x-at}^{x+at} \psi(y)\ dy.$$

Show that

(i) $u(x_0, t) = 0\ \ \forall t \geq 0$, if φ, ψ are odd with respect to x_0;

(ii) $u_x(x_0, t) = 0\ \ \forall t \geq 0$, if φ, ψ are even with respect to x_0.

(*This is Lemma 1.2.8 of the lecture.*)

Exercise 9:

Determine the general solutions u of the following PDEs using some suitable coordinate transformation:

a) $12u_x(x,y) + 4u_y(x,y) = x$;

b) $9u_{xx}(x,y) - \frac{1}{4}u_{yy}(x,y) = \sin(x)$;

c) $2u_x(x,y,z) + 3u_y(x,y,z) + 4u_z(x,y,z) = e^{x+y+z}$.