Prof. Dr. J. Sprekels
Humboldt-Universität zu Berlin
Institut für Mathematik

"Partial Differential Equations" (BMS)
– SS 10 –
Exercises No. 4
to be presented on May 18, 2010

Exercise 10:

Solve the following first order PDEs using characteristics:

a) $u_t + bD_x u = f$ in $\mathbb{R}^n \times \mathbb{R}^+$ [where $b \in \mathbb{R}^n$, $f \in C(\mathbb{R}^{n+1})$ are given]
subject to the boundary condition
$u(x,0) = g(x)$, i.e. $u = g$ on $\mathbb{R}^n \times \{0\}$ [with given $g \in C^1(\mathbb{R}^{n+1})$];

b) $xu_x + yu_y = 2u$ in $\mathbb{R}^2$ subject to $u(x,1) = g(x)$;

c) $uu_x + u_y = 1$ in $\mathbb{R}^2$ subject to $u(x,x) = \frac{x}{2}$;

d) $xu_x + 2yu_y + u_t = 3u$ in $\mathbb{R}^3$ subject to $u(x,y,0) = g(x,y)$.

Exercise 11:

Classify the second order PDE (where $u : \mathbb{R}^3 \to \mathbb{R}$):

$$3u_{x_1x_1} + 2u_{x_2x_2} + 6u_{x_1x_2} - 2u_{x_1x_3} - 4u_{x_2x_3} = 0.$$

(*Note that the eigenvalues of the associated matrix can be determined only numerically. So find indirect ways to solve the problem.*)