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"Partial Differential Equations" (BMS)
– SS 10 –
Exercises No. 9

to be presented on June 22, 2010

Exercise 25:

For the given function $f \in L^1_{\text{loc}}(0,2)$ calculate it's first weak derivative $Df \in L^1_{\text{loc}}(0,2)$ (or show that it does't exist).

a) $f(x) := \begin{cases} x, & 0 < x \leq 1 \\ 1, & 1 \leq x < 2 \end{cases};$

b) $f(x) := \begin{cases} x, & 0 < x \leq 1 \\ 2, & 1 < x < 2 \end{cases}.$

Exercise 26:

Let $\Omega \subset \mathbb{R}^n$ be an open set, $f \in L^1_{\text{loc}}(\Omega)$, $\bar{x} \in \Omega$ and put $d := \text{dist}(\bar{x}, \partial\Omega)$. Further let $\omega(x) := \chi_{B_1(0)} \cdot c \cdot \exp(-\frac{1}{1-|x|^2})$ be the standard mollifier and $\omega_h(x) := h^{-n}\omega(\frac{x}{h})$ for $h > 0$.
Then the following holds for $f_h(x) := \int_{\mathbb{R}^n} \omega_h(x-y)f(y)\,dy$ with $d > h > 0$:

a) ${f_h}_{|_{B_{d-h}(\bar{x})}} \in C^\infty(B_{d-h}(\bar{x}))$;

b) $\frac{\partial f_h}{\partial x_j}(x) = \int_\Omega \frac{\partial \omega_h}{\partial x_j}(x-y)f(y)\,dy$ for all $x \in B_{d-h}(\bar{x})$.

(*This are the missing parts of Lemma 5.1.3 of the lecture.*)

Exercise 27:

Use Gauss's Integral Theorem to prove that $\mathring{W}^{1,p}(\Omega) \neq W^{1,p}(\Omega)$ $(1 \leq p < \infty)$ for an open subset $\Omega \subsetneq \mathbb{R}^n$ with Lipschitz boundary.