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"Partial Differential Equations" (BMS)
– SS 10 –
Exercises No. 11

to be presented on July 6, 2010

Exercise 30:

Let $\Omega \subset \mathbb{R}^n$ be open and bounded, $f \in L^2(\Omega)$ and $\mu : \Omega \to \mathbb{R}$ some function such that there is a constant $M > 0$ with $|\mu(x)| \leq M$ for every $x \in \Omega$.
Consider the Dirichlet problem

$$(*) \qquad \begin{aligned} \Delta u + \mu u &= f \qquad \text{in } \Omega \\ u &= 0 \qquad \text{on } \partial\Omega. \end{aligned}$$

Find a condition for M such that $(*)$ has a unique weak solution.

Exercise 31:

Let $\Omega \subset \mathbb{R}^n$ be open and bounded. Show that the canonical embedding $L^2(\Omega) \hookrightarrow H^{-1}(\Omega)$ (cf. script p. 95) is actually compact.