Homework 2

Topology II

Winter 2016/17

Problem 1 (Chain homotopic complexes)
(a) We consider the chain complex

$$0 \to \mathbb{Z} \to \mathbb{Z} \oplus \mathbb{Z} \to \mathbb{Z} \to 0,$$

where the maps are given by $t \mapsto \begin{pmatrix} a \\ b \end{pmatrix} t$ and $\begin{pmatrix} x \\ y \end{pmatrix} \mapsto cx + dy$. For which integers a, b, c, d is this complex chain homotopy equivalent to the trivial complex?

(b) Show that the complexes $0 \to \mathbb{Z} \xrightarrow{\cdot 2} \mathbb{Z} \to 0$ and $0 \to 0 \to \mathbb{Z}/2\mathbb{Z} \to 0$ are not chain homotopy equivalent.

Problem 2 (Diagram chasing) Study the construction of the long exact sequence of the homologies of the chain complexes of abelean groups whose chain groups fit in a short exact sequence. Define the connecting homomorphism by diagram chasing using the exactness of the short sequence.

Discuss the exactness at each of the groups of the long exact sequence.

Problem 3(Hurewicz map)
Show that Hurewicz Map, which assigns to a loop based in a point p of a topological space X a 1-cycle gives a well-defined homomorphism

$$\pi_1(X, p) \longrightarrow H_1(X)$$

which descends to an isomorphism

$$\pi_1(X, p)/[\pi_1(X, p), \pi_1(X, p)] \cong H_1(X).$$

where $[G, G]$ denotes the commutator subgroup of G, i.e. the quotient the abeleanization of $\pi_1(X, p)$.

Problem 4
Compute $H_*(D^2, S^1)$.