# Homework 5

## Topology II

### Winter 2016/17

Please, also have a look at Homework Set 3, since we have not discussed most of them.

**Problem 1**
In a proposition discussed in class we show that for $k < n$ we have

$$\tilde{H}_j(S^n \setminus h(S^k)) \cong \mathbb{Z}$$

for $j = n - k - 1$ and 0 otherwise. With $S^k = D^k_+ \cup D^k_-$ with $D^k_\pm$ being the north and south hemisphere, respectively. Let $S^{k-1} = D^k_+ \cap D^k_-$ be the equator sphere. Then we have a long exact sequence from Mayer-Vietoris

$$0 = \tilde{H}_j(S^n \setminus h(D^k_+)) \oplus \tilde{H}_j(S^n \setminus h(D^k_+)) \longrightarrow \tilde{H}_j(S^n \setminus h(S^{k-1})) \\ \longrightarrow \tilde{H}_{j-1}(S^n \setminus h(S^k)) \longrightarrow \tilde{H}_{j-1}(S^n \setminus h(D^k_+)) \oplus \tilde{H}_{j-1}(S^n \setminus h(D^k_-)) = 0$$

from which follows for $1 \leq k < n$ that

$$\tilde{H}_j(S^n \setminus h(S^{k-1})) \cong \tilde{H}_{j-1}(S^n \setminus h(S^k)).$$

What is wrong with extending the formalism to the case $k = n$? We would get

$$0 = \tilde{H}_0(S^n \setminus h(D^n_+)) \oplus \tilde{H}_0(S^n \setminus h(D^n_-)) \longrightarrow \tilde{H}_0(S^n \setminus h(S^{n-1})) \longrightarrow 0$$

which is wrong since by the proposition

$$\tilde{H}_0(S^n \setminus h(S^{n-1})) \cong \mathbb{Z}!$$

How can we correct the method? What would be non-trivial conclusions? See Hatcher, pg. 170.