51. Find Borel subgroups in $\text{SO}_4$, $\text{Sp}_4$, $T_4$ and $U_4$.

52. Find all parabolic subgroups $P$ with $T_4 \subseteq P \subseteq \text{GL}_4$.

53. Let the connected linear algebraic group $G$ act on a quasi-projective variety $X$ with finitely many orbits. Show that every irreducible closed $G$-invariant subset in $X$ is the closure of a $G$-orbit. Find a counterexample for an action with infinitely many orbits.

54. Find a connected linear algebraic group $G$ and a maximal solvable subgroup $U \subset G$ such that $U$ is disconnected.

55. Classify all root systems in the Euclidean plane $E := \mathbb{R}^2$.

(Note: A root system in an Euclidean space $(E, (-,-))$ is a subset $\Phi \subset E$ such that

(RS1) $\Phi$ is finite, spans $E$ and $0 \notin \Phi$;
(RS2) for any $\alpha \in \Phi$, $\mathbb{R} \alpha \cap \Phi = \{\alpha, -\alpha\}$;
(RS3) for any $\alpha \in \Phi$, the reflection $s_\alpha: E \to E, x \mapsto x - \frac{2(x, \alpha)}{(\alpha, \alpha)} x$ preserves $\Phi$;
(RS4) for any $\alpha, \beta \in \Phi$: $\langle \beta, \alpha \rangle := \frac{2(\beta, \alpha)}{(\alpha, \alpha)} \in \mathbb{Z}$. )

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Lectures: Monday, 12.15, large lecture hall Wegelerstraße 10
Thursday, 14.15, small lecture hall Wegelerstraße 10

Tutorials: Wednesday, 16.15 (Orlando); Friday, 12.30 (Tomasz)