Addendum to II of "Corrections + Remarks"

We were rather careless in our treatment of the "radical anomaly" defined on p. 24, 25 of II. The correction was devised by Martin Zeman. We again let $\langle W, \mathbf{w}, \mathbf{w}' \rangle$ witness the goodness of $\langle M, W, \mathbf{w} \rangle$ and let $k$ be as on p. 23. Let $S$ be an iteration strategy for $M$ and $\mathcal{S}$ the derived iteration strategy for $\langle M, W, \mathbf{w} \rangle$ as in p. 23. We again continue $\langle M, W, \mathbf{w} \rangle$ against $M$, using $S, S$. Let $Y^W = \langle \langle W_i \rangle, \ldots, T^W \rangle$ again be the iterates of $\langle M, W, \mathbf{w} \rangle$ and $Y^Z = \langle \langle Z_i \rangle, \ldots, T^Z \rangle$ be the iteration on the $M$ side. Let $Y^M = \xi(Y^W) = \langle \langle M_i \rangle, \ldots, T^M \rangle$ again be the $\langle k, k \rangle$-copy of $Y^W$ with copying maps $\langle \xi_i \rangle$. As before, we call $i = 3 + 1$ a radical anomaly if $\mathbb{E}_3^W = \mathbb{E}$ is a superstrong index in $M$. (Here $\mathbb{E}_3 = \mathbb{E}_3^W + \mathbb{E}_3^W$, where $\mathbb{E}_3^W = \xi \mathbb{E}_3^W$.) Let $i$ be such an anomaly. Then $W_i = \langle \mathbb{E}_3^W, F \rangle$, where $\mathbb{E}_3^W$, $M: \mathbb{E}_3^W \rightarrow \mathbb{E}_3^W$. 
$F \neq \emptyset$, since $\mathcal{M} \mathcal{N} \mathcal{X} = \langle \nu_\gamma^{\mathcal{E}_y}, F \rangle$ and $F \neq \emptyset$. But $E_\gamma = \emptyset$, since $\nu_\gamma$ is a comparison iteration. Hence $\nu_\gamma = \nu_\gamma^T$, where $\gamma < i$. Thus the iteration $\mathcal{M} \mathcal{N} \mathcal{X}$ is not normal and our iteration strategy $S$ is not directly applicable to $\mathcal{M} \mathcal{N} \mathcal{X}$. We get around this by defining a modified iteration

$\mathcal{M}^{M'} = \langle \{M_i'\}, \{\nu_i; i \in \mathcal{O}'\}, \{\gamma_i\}, \{\pi_i'\}, T' \rangle$

with the properties:

(i) $\mathcal{M}^{M'}$ is normal.

(ii) $M_i = M_i'$ unless $M_i$ is a radical anomaly.

(iii) If $M_i = M_i'$, $M_i = M_i'$. Then $i T_i' \leftrightarrow i T_i$ and $\pi_i = \pi_i'$.

(iv) If $i$ is a radical anomaly, then $T'(i) = -1$.

Next let $b'$ be a branch of limit length through $\mathcal{M}^{M'}$. There is at most one radical anomaly on $b'$ by (iv). Hence $b = \{h| (M_i' = M_i \land i \in b \land h T_i)\}$ is a branch through $\mathcal{M}^{M'}$ with $M_b = M_b'$. Hence at limit points $\lambda$, we apply $S$ to $\mathcal{M}^{M'}_\lambda$ to get $M'_\lambda = M_\lambda$. This gives us a strategy for $\mathcal{M}^{M'}$ and
The derived strategy for $\gamma^w$. Our first observation is that if $i = 3+1$ is a radical anomaly and $F = E_i^{m'}$, $F' = E_i^{w'}$ are as above. Then $i \neq T(i + 1)$ for all $i \geq i$, since $\lambda = \lambda_i^3$ and $T(i + 1)$ is the least $t$ s.t. $\lambda_t < \lambda_i^3$. Thus $i \neq T(i)$ for $i < i$. We have:

$$\text{crit}(E_i^{m'}) = \text{crit}(F) < \lambda,$$

where $\lambda$ = the largest cardinal $< \theta'$ in $M$. But $\text{crit}(E_i^{m'}) = \lambda$. Hence $\lambda$ cannot be an anomaly. Now let $\gamma^{m'} | i$ be defined. Define $\gamma^{m'} | i + 1$ by setting $M'_{i+1} = M_i^{m'} + 1$ (i.e. $\gamma^{m'} | i + 1 : M \rightarrow M'_{i+1}$), where

$$F' = E_i^{m'} = \overline{\nu}_{i-\sigma}^{m'}(F).$$

Then

$$F' = E_i^{m'} = \overline{\nu}_{i-\sigma}^{m'}(F) \leq \nu_i^{m'},$$

$$\nu_i^{m'} = \nu_i^{m'}(\lambda + \theta') \leq \nu_3^{m'} = \overline{\nu}_{i-\sigma}^{m'}(\lambda') \leq \nu_3^{m'}(\lambda + \theta'),$$

$$\nu_3^{m'} = \nu_3^{m'}(\lambda + \theta').$$

We then set:

$$\nu_3^{m'} = \nu_3^{m'}(\lambda + \theta').$$

This defines $\gamma^{m'} | i + 1$, which...
in easily seen to be normal. We define $\gamma M'_i i + 2$ by nothing; if $D'$ (then $M'_i = M'_{i + 1}$, $\mu_i, i + 1 = 10$). Finally, if $j$ is neither an anomaly or the
successor of an anomaly, then we set $\lambda M'_i = M'_j$. At $j = h + 1$, we set $\lambda h = \lambda h$. This defines
$\gamma M'_{i + 1}$. The inductive verification of the properties (i) - (iv)
is straightforward. (For $j = h + 1$ we observe that $\overline{3} = T(j)$ is not an anomaly. We also have $\lambda \overline{3} = \lambda \overline{3}$,
even if $\overline{3} + 1$ is an anomaly.
Hence $T'(i) = \overline{3}$ and $\overline{M'}_{\overline{3}} = \overline{M'}_{\overline{3}}$.

A second problem remains.
It can be shown as before that the
concatenation $<\gamma W, \gamma W>$ terminates (Lemma. The proof of Lemma 2 (p. 28) must be
amended; however, a mistake occurs in the proof of Case 2.1.5,}
The proofs of (11), (12) go through in the same way as the proofs of \( i \neq j \) and \( i \neq k \). A new argument is needed, however, to show \( j \neq i \). (This, too, is due to Martin Zeman.) Suppose not.

Then \( E_{W, \xi} \) fails to satisfy the initial segment condition, since otherwise we could repeat the argument for \( i \neq j \). This can only happen if some \( i' \leq \tau_W \) is a cardinal anomaly. But then \( i = i' = \xi + 1 \) and \( E_{W, \xi} \) is the top extender. This means, in particular, that \( \prod_{\xi} \langle M, \xi, E_{W, \xi} \rangle \) and hence \( E^M_{\xi} \neq \emptyset \). But \( \xi \) is a cardinal in \( W, i \); hence, \( E^W_{\xi} = \emptyset \). However \( \nu_0 = \xi^+ \) and \( E^M_{\nu_0} \neq \emptyset \).

But we cannot have \( \nu_1 > \nu_0 \), since then \( E^Q_{\nu_1} \) would fail to satisfy the initial segment condition (since then \( \langle \nu_1, E^Q_{\nu_1}, E_{\xi} \rangle \) is a premouse and \( \nu_0 \) is a cardinal in \( W, i \)). Hence \( \nu_1 = \nu_0 \).
and \( j = 0 \), since \( \delta_1 i \in D_1 Q \) and \( y_1 Q \) is a normal iteration. Thus \( \Theta \equiv I_1 Q \). Let \( y_1 i \Theta \equiv I \). Let \( z \in \mathbb{R}^k \) be n.t. \( 1 = T^i \). Let \( x \in \mathbb{R}^k \). Then \( x = T^i \), \( f(x) \), \( x \in \mathbb{R}^k \), \( f \in \mathbb{R}^k \), \( f \in M \), and \( x = \text{crit}(E_{\theta} \mathbb{R}^k) \). But \( \pi_{I_1, \Theta} Q(f) = \pi_{I_1, \Theta} ^{T^i} (f) = \pi_{I_1, \Theta} ^{T^i} \). Hence \( \pi_{I_1, \Theta} ^{T^i} (x) = \pi_{I_1, \Theta} ^{T^i} (f(x)) = \pi_{I_1, \Theta} ^{T^i} (\pi_{I_1, \Theta} ^{T^i} (f(x))) = \pi_{I_1, \Theta} ^{T^i} (x) \). Hence for \( \beta < \lambda \), \( \beta \in E_{\Theta} \mathbb{R}^k \) \( \Rightarrow \beta \in \pi_{I_1, \Theta} ^{T^i} (x) \) \( \Leftrightarrow \beta \in \pi_{I_1, \Theta} ^{T^i} (f(x)) \)

and for \( \beta < \lambda \), \( \beta \in E_{\Theta} \mathbb{R}^k \) \( \Rightarrow \beta \in \pi_{I_1, \Theta} ^{T^i} (x) \) \( \Leftrightarrow \beta \in \pi_{I_1, \Theta} ^{T^i} (f(x)) \)

Since \( E_{\Theta} \mathbb{R}^k \) \( \equiv \Theta \), it satisfies the initial segment condition, it follows from the previous argument that each of the cases \( k = 5 \), \( 3 \leq k \), \( k \geq 5 \) yields a contradiction.

\( \blacksquare \)