Thus, every weakly universal weasel is universal.

Proof:
Let $W$ be a counterexample. Let $N$ be a coiterable pre-mouse whose coiteration does not terminate.
Let $W_i, N_i$ be the coiteration with indices $N_i, W_i$ ($N_i = W_i + Wi$).
Let $C$ be an $\mathcal{E}$-closed set such that $\overline{N}_iW_i(N_i) = N_i$ for $i, j \in C, i \leq j$. For $i \in C$ set $\overline{N}_i = N_iW_i(N_i)$. Then:

1. $\langle W_i, j \geq i \rangle$, $\langle \overline{N}_i, j \geq i \rangle$ is the coiteration of $W_i, \overline{N}_i$ for $i \in C$.

Hence:

2. $\overline{N}_i$ is not a mouse.

We shall derive a contradiction by showing that some $\overline{N}_i$ is a mouse.
Set: $W' = W_0$. Let $D$ be the canonical $w$-complete we use.

Concretely, $W', D$ to $W'', D'$.

Let $W'_i, D_i$ be the concretions with induction $\bar{D}_i, \bar{W}_i$, $\bar{D}_i = \bar{D}_i + \bar{W}_i$.

Set: $\bar{W}_i = \begin{cases} W' & \text{if defined} \\ W'' & \text{if not} \end{cases}$

(similarly for $\bar{D}_i$). Let $D$ be the set of $w$ net.

(a) $\kappa = \sup \{ \bar{W}_i \mid i < \kappa \land \bar{W}_i \text{ defined} \}$

(b) $\prod W'_{\bar{W}_i} (\kappa) = \prod D_{\bar{D}_i} (\kappa) = \kappa$

(c) $\sup (\kappa) > \omega$.

For $\kappa \in D$, let $\nu = \kappa + \bar{W}_\kappa$ and define $\bar{N}_\kappa = \bigcup_{\nu} E_{W'_{\bar{W}_\kappa}}(U_\kappa)$ by

(3) $\prod W'_{\bar{W}_\kappa} (\bar{N}_\kappa) : \bar{N}_\kappa \to E_0 \bar{N}_\kappa$

confinally.

We know that $E_{\nu_{\bar{N}_\kappa}}(\kappa)$ is $\omega$-complete and, in fact, that
(4) \( X \in E^\bar{N}_\nu \) if \( \nu_i \in X \) for sufficiently large \( i < \nu \).

By the proof of § 3.1 Lemma 2.3 it follows that \( U_\nu \) is \( \omega \)-complete and that

(5) \( X \in U_\nu \) if \( \bar{w}_i \bar{w}_j (\nu_i) \in X \) for sufficiently large \( i < \nu \).

A straightforward repetition of the proof of § 3.1 Lemma 2.7 shows that

(6) \( U_\nu = E^{\bar{Q}_\nu} \) for some \( \nu \in D \).

Fix \( \nu \). Then:

(7) \( \bar{N}_\nu \) is a monic

(8) \( \bar{p}_\nu = \nu \).

Using this we prove:

Claim 11: \( \bar{N}_\nu \rightarrow \bar{N}_\nu \).
where $\pi = \pi_{W, \tilde{W}, \tilde{N}} \upharpoonright \tilde{N}_\kappa$, which establishes that $\tilde{N}_\kappa$ is a model, contradicting (2).

(9) \[ A \models p^\kappa \] Then $\pi : \tilde{N}_\kappa \to \tilde{N}_\kappa$.

Proof. (Ind. on \( n \)).

$n = 0$ trivial. Let it hold for \( n \).

Let $p \in \pi^{N}_\kappa$, $\tilde{B} = A^{\pi, p}_\kappa$, \( \tilde{B} = A^{\pi, p}_\kappa \).

We claim: \[ \pi(\tilde{B}) \]

\[ \pi : \langle \tilde{N}_\kappa, \tilde{B} \rangle \to \langle \tilde{N}_\kappa, \tilde{B} \rangle. \]

Let $\tilde{A} = A^{\pi, p}_\kappa \tilde{N}_\kappa$. Then \( \tilde{A} = A^{\pi, p}_\kappa \tilde{N}_\kappa \).

Set: $B = \bigcup_{\tilde{x} \in \tilde{N}_\kappa} \tilde{B} \cap \tilde{x}$. Then

(10) \[ \pi : \langle \tilde{N}_\kappa, \tilde{A}, \tilde{B} \rangle \to \langle \tilde{N}_\kappa, \tilde{A}, B \rangle \]

and it suffices to show

that $B = \tilde{B}$. $\tilde{B}$ is uniformly definable from $\tilde{A}, p$, by a $\Sigma_1$ formula $\forall y \phi$ and we must show that $B$ is defined from $\tilde{A}, \pi(p)$.
by the same formula, i.e.,
\[ z \in B \implies \langle \tilde{N}_n, \tilde{A} \rangle = \forall y \forall \pi [z, \pi(p_n)] \]
for \( z \in \tilde{N}_n \).

\((\Leftarrow) \) in \( \Delta_1(\langle \tilde{N}_n, \tilde{A}, B \rangle) \) + hence follows by \((10)\). To see \((\Rightarrow)\), let \( z \in B \) + suppose that
\( z \in \pi(w) \) for a \( u \in \tilde{N}_n \). Note
That \( \langle \tilde{N}_n, \tilde{A} \rangle \) is admissible, and so \( \rho_{\tilde{N}_n}^\omega \geq \kappa + \kappa \) is the largest cardinal in \( \tilde{N}_n \). Since

\[ \forall z \in B \exists u \in \langle \tilde{N}_n, \tilde{A} \rangle = \forall y \forall \pi [z, \pi(p_n)] \]

there is \( w \in \tilde{N}_n \) such

\[ \forall z \in B \exists u \in \pi(w) \langle \tilde{N}_n, \tilde{A} \rangle = \forall y \forall \pi [z, \pi(p_n)] \]

Hence by \((10)\)

\[ \forall z \in B \forall \pi(w) \exists u \langle \tilde{N}_n, \tilde{A} \rangle = \forall y \forall \pi [z, \pi(p_n)] \]

QED\((91)\)

The claim then follows by:

\[ (11) \rho_{\tilde{N}_n}^\omega \geq \kappa \]

since otherwise, if \( n \in \)
least \( n \cdot t \cdot \frac{p_{m+1}}{N_n} < \nu_n \), then

\( h_{\tilde{N}_n} \) is cofinal in \( \nu_n \).

It follows by \((91)\) that

\( h_{\tilde{N}_n} \) is cofinal in \( \nu \).

Hence \( \frac{p_{m+1}}{N_n} < \nu \). Contrad!

QED