Appendix to §8

We again attempt to carry out our proof for arbitrary mice; using
Neeman–Steel in place of Dodd–Jensen. Let $M$ be a countable mouse
and $\mathcal{S} = \langle \mathcal{S}_i : i < \omega \rangle$ an enumeration
of $M$, we say that an iteration strategy in $\mathcal{S}$ is \underline{minimal} if it
has the properties given by the Neeman–Steel lemma. We say
that $S$ is \underline{minimal} if it is
$\mathcal{S}$-minimal for some $\mathcal{S}$. Now
let $\sigma : W \to M$ witness the goodness
of $\langle M, W_i \rangle$. We say that
a coiteration $\langle yW, yM \rangle$ of $\langle M, W_i \rangle$ against $M$ is \underline{minimal}
iff the copy $\sigma(yW)$ exists and
$\sigma(yW), yM$ employ the same
minimal strategy. Lemma 1 then
holds in the obvious reformulation:
Lemma 1. Let $M$ be a countable mouse.

Let $o : W \rightarrow M$ witness the goodness of $\langle M, W, \lambda \rangle$. Let $\langle y^W, y^Q \rangle$ be a minimal coiteration of $\langle M, W, \lambda \rangle$ against $M$. Let $\Theta = \|y^Q\|$. Then:

(a) $\Theta \geq 0$ in $y^W$.
(b) $W_\Theta$ is a simple iterate of $W$ in $y^W$ and a segment of $Q_\Theta$.
(c) $W$ is a mouse.

The proof of Lemma 2 goes through as before: by Löwenheim–Skolem we take $M$ as countable. The double rooted coiteration in the proof is then taken as minimal. Lemmas 3.1, 3.2 hold as before. In the proof of Lemma 4 we again (w.l.o.g.) take $M$ as countable and apply a minimal double rooted coiteration. (We induct on the relation $\lambda$.)
$\text{N} \notin \text{N}'$ iff $\text{N}$ is a countable non-simple iterate of $\text{N}'$. ) At two places,
however, we used a property of mice which was derived from
basiness:

\[ (*) \text{ Let } \text{N} = \langle \mathcal{J}^\mathcal{E}, \mathcal{F} \rangle \text{ be a mouse with } \mathcal{F} \neq \emptyset. \text{ Then } \omega \mathcal{P}^\mathcal{E} < \mathcal{N} = \text{ the largest cardinal in } \text{N}. \]

This was used in Case 2.2 (to prove that $\nu_0 > \text{unf} \text{N}$ ) and in (1) of Case 2.3.

Thus, to carry out the proof, we must assume:

\[ (**) \text{ Every segment of } \text{N} \text{satisfies } \]

There is no doubt that we can go very far using only mice
which satisfy (**) . The existence
of a counterexample to (**) has, in fact, not been shown,
even using very large cardinals.

However, even if we drop (**) ,
we can still get a weaker version of Lemma 4. (c) must then be replaced by:

(c') $M$ is a segment of $H$, where $\pi : M \rightarrow H$ for some $\gamma \in M$, $\mu \leq \gamma$.

The weakening is necessary only if $\nu > \omega \cdot \eta M$ fails in Case 2.2 or (1) of Case 2.3 fails. If that happens, we still get (c') with $\gamma = \mu = \nu$ and $M$ is a proper segment of $H$. (We omit the proof.) But then $\omega \cdot M \geq \nu$, since $\nu$ is a cardinal in $H$.

Thus the full conclusion of Lemma 4 will still hold if $\omega \cdot M < \nu$. 
The proof of Lemma 4.1 goes through as before with the obvious modifications. However, we need §6 Cor 7.1, which was a strong consequence of basicness. Hence Lemma 4.1 is proven only for mice $M$ all of whose segments satisfy §6 Cor 7.1. The proof of Cor 4.2 goes through for arbitrary mice, Cor 4.3 goes through for $M$ satisfying (4) (4.2 and 4.3 show how difficult it will be to find nice non-satisfying (4)). Lemma 5 goes through in full generality.
The proof of Lemma 4.1 goes through as before with the obvious modifications. However, we need §6 Cor 7.1, which was a strong consequence of baricness. Hence Lemma 4.1 is proven only for mice $M$ all of whose segments satisfy §6 Cor 7.1. The proof of Cor 4.2 goes through for arbitrary mice. Cor 4.3 goes thru for $M$ satisfying $(\ast)$. (4.2 and 4.3 show how difficult it will be to find mice not satisfying $(\ast)$). Lemma 5 goes through in full generality.