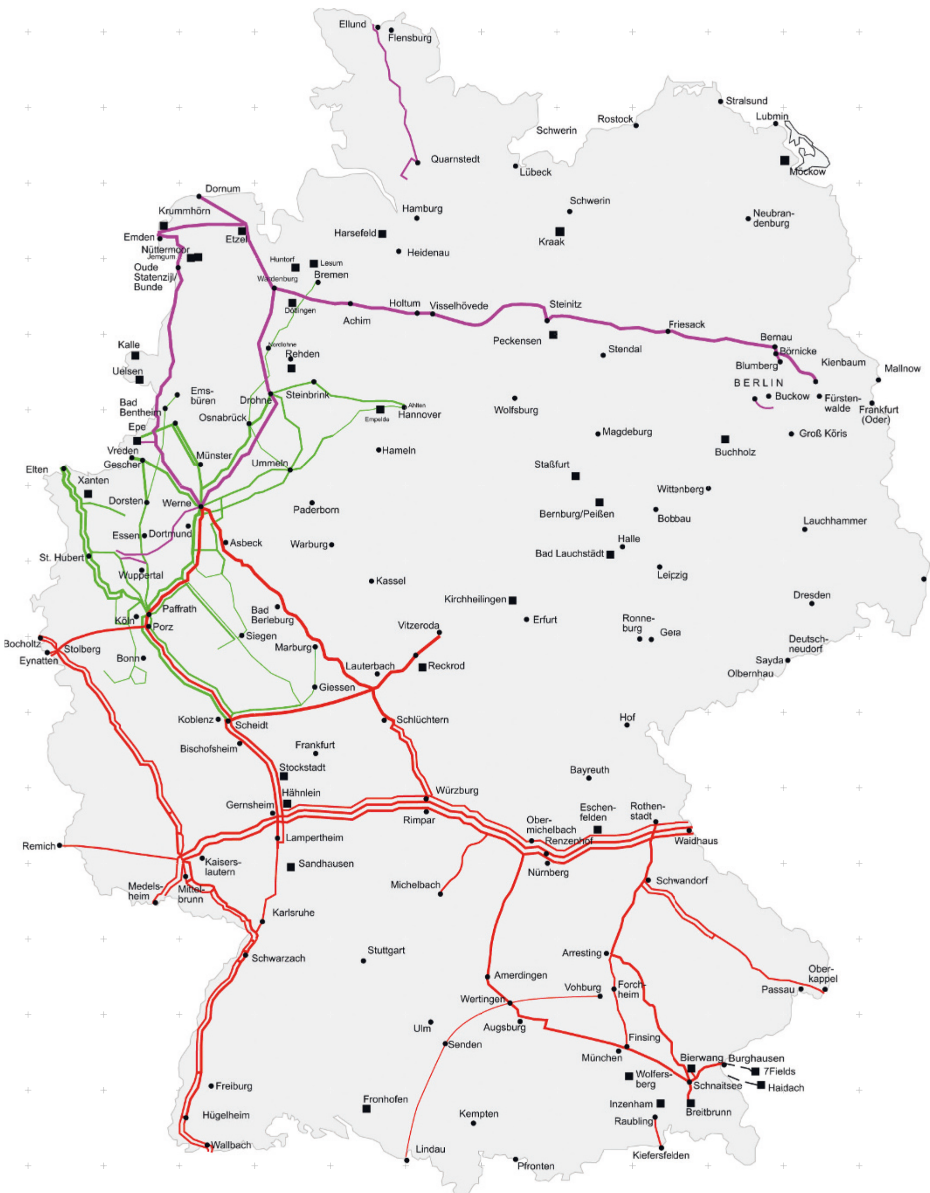


Network systems of Open Grid Europe. The network model is split into three parts: H-gas north (purple), H-gas south (red), and L-gas (green). (Source: Open Grid Europe.)


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# CAPACITY EVALUATION FOR LARGE-SCALE GAS NETWORKS

## A discrete-continuous model for optimal transport of gas

Natural gas is important for the energy turnaround in many countries like in Germany, where it serves as a bridging energy towards a fossil-free energy supply in the future. About 20% of the total German energy demand is provided by natural gas, which is transported through a complex pipeline network with a total length of about 30000 km, and the efficient use of the given transport infrastructure for natural gas is of political, economic, and societal importance. As a consequence of the liberalization of the European gas market in the last decades, gas trading and transport have been decoupled. This has led to new challenges for gas transport companies, and mathematical optimization is perfectly suited for tackling many of these challenges. However, the underlying mathematical problems are by far too hard to be solved by today's general-purpose software so that novel mathematical theory and algorithms are needed. The industrial research project 'ForNe: Research Cooperation Network Optimization' has been initiated and funded by Open Grid Europe in 2009 and brought together experts in mathematical optimization from seven German universities and research institutes, which cover almost the entire range of mathematical optimization: integer and nonlinear optimization as well as optimization under uncertainty. The mathematical research results have been put together in a software package that has been delivered to Open Grid Europe at the end of the project. Moreover, the research is still continuing – e.g., in the Collaborative Research Center/Transregio 154 „Mathematical Modelling, Simulation and Optimization using the Example of Gas Networks" funded by the German Research Foundation.


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## Industrial challenge and motivation

Since the liberalization of the gas market in Europe by the EU Commission from 1998 on, trade and transport of natural gas are decoupled and have to be conducted by separate companies. Today, the European transmission system operators (TSOs) usually operate under the so-called entry-exit system [1, 5]. A brief explanation of this system is as follows. First, the TSOs publish technical capacities for all the points in their network at which gas can be supplied or withdrawn. Second, gas traders can then book a capacity right at these nodes up to the previously published technical capacity of the node. This booking serves as a mid- to long-term capacity-right contract. Third, on a day-to-day basis, these traders can nominate a certain amount up to their booked capacity and only have to ensure that all their nominations are in balance, i.e., the total amount of supplied gas has to match the total amount of withdrawn gas. Fourth and lastly, the TSO has to transport the actual nomination.

The main complexity lies in the fact that the TSOs have to be able to transport any balanced nomination that is in compliance with the respective bookings. Thus, the feasibility of transport needs, in principle, to be checked for an infinite number of possible nominations. This is in contrast to the goal of the TSOs to publish technical capacities for possible bookings that are as large as possible. Consequently, for an efficient usage of resources and long-term planning, it is important to compute the capacities of the network.

There are two major challenges for TSOs like Open Grid Europe: First, the usage of capacities by the gas traders is uncertain, i.e., it is unknown which nominations will arise. Second, deciding whether a nomination can be operated through the network involves both integer and highly nonlinear aspects. The task is thus properly modeled as a stochastic mixed-integer nonlinear optimization problem. Since these problems need to be solved on complex and large-scale transport networks, there was a strong need for new mathematical techniques for solving these challenging optimization problems.

Thus, Open Grid Europe formed an industrial research project that included seven universities and research institutions in Germany, covering the range of integer, stochastic, and nonlinear optimization.

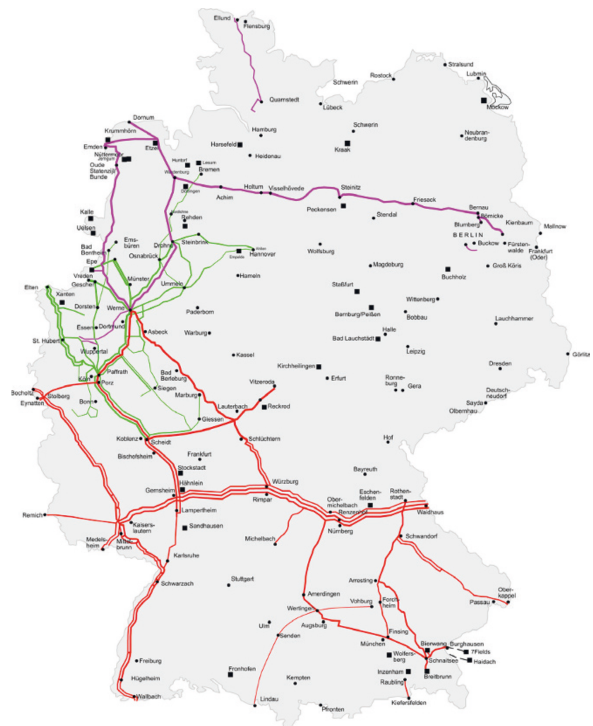

**Figure 1:** The natural gas transport network of Open Grid Europe. The mathematical optimization model for the optimal transport of gas through such a network combines almost all challenges of modern optimization: integer controls, differential equations, highly nonlinear models of physics and engineering, as well as uncertain data.

## Mathematical research

Since by signing a booking contract the TSO guarantees to transport any possible load flow situation (that complies to this booking), checking the feasibility of nominations lies at the heart of the research project. A rigorous mathematical modeling of gas transport needs to consider different elements of a gas transport network. Pipes are outnumbering all other devices in these networks. The flow of natural gas through a pipe can be modeled by using the Euler equations of compressible fluids in cylindrical pipes:

$$\frac{\partial \rho}{\partial t} + \frac{1}{A}\frac{\partial q}{\partial x} = 0,$$
$$\frac{1}{A}\frac{\partial q}{\partial t} + \frac{\partial p}{\partial x} + \frac{1}{A}\frac{\partial (qv)}{\partial x} = -\lambda(q)\frac{|v|v}{2D}\rho - g\rho h'.$$

Together with a suitable equation of state, e.g., $p = \rho R_s T z$, this system of nonlinear and hyperbolic partial differential equations (PDEs) couples the main physical quantities mass flow $q$, pressure $p$, temperature $T$, velocity $v$, and density $\rho$ in dependence of the parameters diameter $D$, cross-sectional area $A$, friction $\lambda$, gravitational acceleration $g$, the pipe's slope $h'$, the specific gas constant $R_s$, and the compressibility factor $z$. Since the main focus of the research project was on planning problems instead of on operational control of the network, it is reasonable to abstract from time-dependent effects, i.e., to consider the stationary variant

$$\frac{\partial q}{\partial x} = 0, \qquad \frac{\partial p}{\partial x} + \frac{1}{A}\frac{\partial (qv)}{\partial x} = -\lambda(q)\frac{|v|v}{2D}\rho - g\rho h'$$

of the above mentioned PDEs. To control the flow through the network and its pressure, other network elements like compressors or control valves need to be operated. The respective models usually are of mixed-integer type and the mathematical problem of cost-optimal gas transport thus translates to a mixed-integer nonlinear optimization problem with ordinary differential equations. The day-to-day practice in planning departments of many TSOs is that the feasibility of a specific nomination is checked by using simulation software. Here, the user needs to configure the controllable elements of the transport network and the simulation tool computes a physical state based on this control, for which feasibility can be checked afterward. In the case of infeasibility, the controls need to be adapted manually and the process is repeated. Here, mathematical optimization comes into play because it allows to automatize this process. Our solution approach for validating the feasibility of a nomination is sketched in the lower block of the flow chart given in Figure 2. Since the original mixed-integer nonlinear problem is by far too hard to be solved on real-world networks, the solution process is split up into two different components. First, a variety of physically simplified models based on mixed-integer linear optimization, mathematical programming with equilibrium constraints, mixed-integer nonlinear optimization, and purely continuous optimization are solved to guess reasonable integer controls of the network. Afterward, the feasibility of these controls are verified by using a detailed nonlinear model that represents physical and technical requirements very precisely.

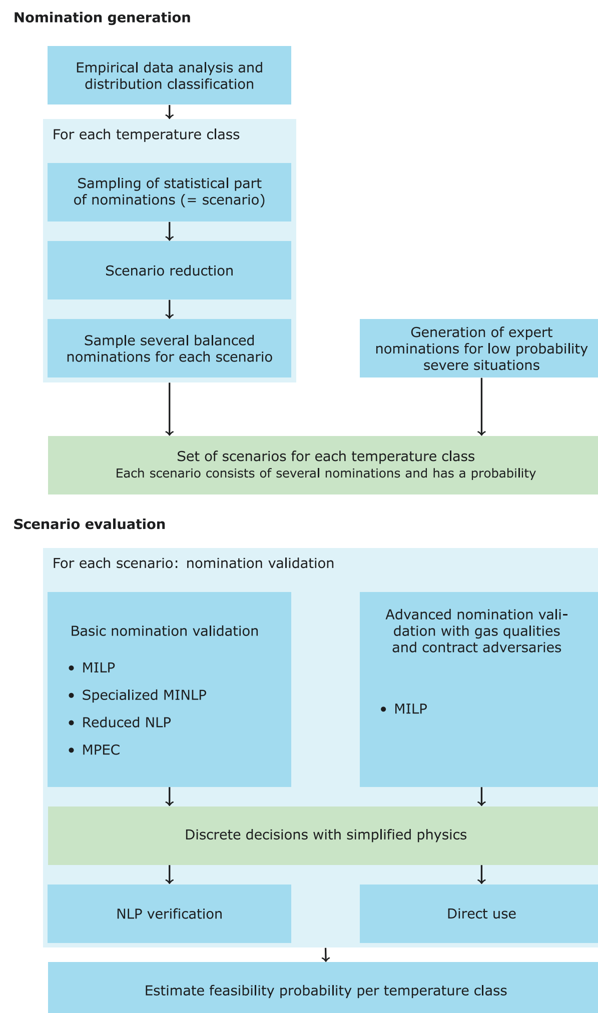


**Figure 2:** Flow chart of the solution approach for validating the feasibility of a booking contract.

On top of that, verifying the feasibility of a booking has to account for all possible nominations that might occur. This requirement needs to be relaxed for practice and is replaced by an approach that guarantees the feasibility of a nomination with a high probability. The respective nomination generation approach is sketched in the upper block of Figure 2. First, by analyzing historical gas flow data from several years, we estimate probability distributions modeling the demands at the exits. This information and the capacity contracts in question are used to sample the space of possible nominations, which are then checked for feasibility by the techniques described above. Finally, the results of thousands of samples are combined and analyzed to give an estimate for the probability of validity of the offered capacities. For the details, we refer to [2, 3, 4, 6].

## Implementation

The research project "ForNe: Research Cooperation Network Optimization"[1] was initiated in 2009 by Open Grid Europe, Germany's largest gas network operator, and conducted jointly by seven research institutes and universities: Zuse Institute Berlin (Mathematical Optimization), Friedrich-Alexander Universität Erlangen-Nürnberg (Discrete Optimization), Leibniz Universität Hannover (Institut für Angewandte Mathematik), Universität Duisburg-Essen (Fakultät für Mathematik), Technische Universität Darmstadt (Fachbereich Mathematik), Humboldt-Universität zu Berlin (Institut für Mathematik), and the Weierstraß-Institut für Angewandte Analysis und Stochastik Berlin.

The success of the project was, in particular, due to regular meetings of the scientific partners with the industrial partner Open Grid Europe that took place multiple times per year. During these meetings, Open Grid Europe not only provided us with the relevant data but the discussions also gave insights into the structure of the studied problems that later helped to develop efficient solution techniques.

Based on these measures, the project contributed a new methodology for this problem, including innovative solution approaches for mixed-integer nonlinear optimization problems under uncertainty. It also triggered the research project "Untersuchung der technischen Kapazität von Gasnetzen", which was funded by the German Federal Ministry of Economic Affairs and Energy (Bundesministerium für Wirtschaft und Energie – BMWi) during the years 2009–2012. Moreover, it has been supported by the Bundesnetzagentur (BNetzA).

[1] `https://www.zib.de/projects/forne-research-cooperation-network-optimization`

## Industrial relevance and summary

A comprehensive software implementing our methods was evaluated on real-world large-scale networks and delivered to Open Grid Europe. In the course of the project, ten PhD theses and more than 20 research articles have been published. The developed methods are comprehensively described in the book "Evaluating gas network capacities" [3], which received the 2016 EURO Excellence in Practice Award. As part of the above mentioned BMWi project, elaborate instance format descriptions and a library of gas network instances were published at `http://gaslib.zib.de`; see also [8]. Moreover, the research results also led to the text book [7] that already served as a basis in applied mathematical optimization lectures at German universities.

The research of this project is also continued in the Collaborative Research Center/Transregio 154 "Mathematical Modelling, Simulation and Optimization using the Example of Gas Networks" funded by the German Research Foundation, which started in 2014 and which is still going on.

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## References

[1] A. Fügenschuh, B. Geißler, R. Gollmer, C. Hayn, R. Henrion, B. Hiller, J. Humpola, T. Koch, T. Lehmann, A. Martin, R. Mirkov, A. Morsi, J. Rövekamp, L. Schewe, M. Schmidt, R. Schultz, R. Schwarz, J. Schweiger, C. Stangl, M. C. Steinbach, and B. M.

Willert. Mathematical optimization for challenging network planning problems in unbundled liberalized gas markets. *Energy Systems*, 5(3):449–473, 2014.

[2] B. Hiller, T. Koch, L. Schewe, R. Schwarz, and J. Schweiger. A system to evaluate gas network capacities: Concepts and implementation. *European Journal of Operational Research*, 270(3):797–808, 2018.

[3] T. Koch, B. Hiller, M. E. Pfetsch, and L. Schewe, editors. *Evaluating Gas Network Capacities*. MOS-SIAM Series on Optimization. SIAM, 2015.

[4] T. Koch, H. Leövey, R. Mirkov, W. Römisch, and I. Wegner-Specht. Szenariogenerierung zur Modellierung der stochastischen Ausspeiselasten in einem Gastransportnetz. In *Optimierung in der Energiewirtschaft*, VDI-Berichte 2157, pages 115–125. VDI-Verlag, Düsseldorf, 2011.

[5] A. Martin, B. Geißler, C. Hayn, A. Morsi, L. Schewe, B. Hiller, J. Humpola, T. Koch, T. Lehmann, R. Schwarz, J. Schweiger, M. Pfetsch, M. Schmidt, M. Steinbach, B. Willert, and R. Schultz. Optimierung Technischer Kapazitäten in Gasnetzen. In *Optimierung in der Energiewirtschaft*, VDI-Berichte 2157, pages 105–114, 2011.

[6] M. E. Pfetsch, A. Fügenschuh, B. Geißler, N. Geißler, R. Gollmer, B. Hiller, J. Humpola, T. Koch, T. Lehmann, A. Martin, A. Morsi, J. Rövekamp, L. Schewe, M. Schmidt, R. Schultz, R. Schwarz, J. Schweiger, C. Stangl, M. C. Steinbach, S. Vigerske, and B. M. Willert. Validation of nominations in gas network optimization: models, methods, and solutions. *Optimization Methods and Software*, 30(1):15–53, 2015.

[7] L. Schewe and M. Schmidt. *Optimierung von Versorgungsnetzen. Mathematische Modellierung und Lösungstechniken*. Springer Spektrum, Berlin, Heidelberg, 2019.

[8] M. Schmidt, D. Aßmann, R. Burlacu, J. Humpola, I. Joormann, N. Kanelakis, T. Koch, D. Oucherif, M. E. Pfetsch, L. Schewe, R. Schwarz, and M. Sirvent. GasLib-a library of gas network instances. *Data*, 2(4), 2017.