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Amoebas of genus at most 1

Abstract

For a complex polynomial $f \in \mathbb{C}[z_1, \ldots, z_n]$ with variety $\mathcal{V}(f) \subset (\mathbb{C}^*)^n$ the *amoeba* of f is the image of $\mathcal{V}(f)$ under the Log-map :

$$\text{Log} : (\mathbb{C}^*)^n \to \mathbb{R}^n, \; \left(|z_1| \cdot e^{i \cdot \phi_1}, \ldots, |z_n| \cdot e^{i \cdot \phi_n}\right) \mapsto (\log|z_1|, \ldots, \log|z_n|)$$

and will be denoted as $\mathcal{A}(f)$. Amoebas are closed sets with non–empty complement . Each complement component of $\mathcal{A}(f)$ is convex and the number of complement components of $\mathcal{A}(f)$ is limited by the number of monomials of f which coincides with the number of lattice points in the Newton polytope $\text{New}(f)$.

The existence of inner complement components of $\mathcal{A}(f)$ (for a fixed f) depends on the choice of the coefficients of f. Two major open questions on amoebas are:

1. How large to choice the modulus $|c|$ of a coefficient c "corresponding" to an inner complement component in relation to the other coefficients to let this complement component exist and, if one has such a boundary,
2. which points of $\mathcal{A}(f)$ switch to the complement if one passes the boundary. I.e.: At which point of the amoeba the complement component will appear.

We will restrict to amoebas of genus 1 (i.e. with at most one inner complement component) with a simplex as Newton polytope and show that one can compute boundaries for the inner complement component to exist and also compute the point $\delta + \nu$ *where* the inner complement component will appear.
This result yields that the configuration space $\{(b_1, \ldots, b_n, c) \mid b_1, \ldots, b_n, c \in \mathbb{C}\}$ of the coefficients of the polynomials f we investigate is seperated in two fulldimensional sets (representing the solid resp. perforated configurations) by an algebraic plane given by the *A–discriminant* $\Delta_A(f)$ of f. Further consequence is some equivalence relation between the existence of an inner complement component and so called *lopsidedness* of the point $\delta + \nu$.

The connection between amoebas and tropical geometry is the *spine* $\mathcal{S}(f)$ of an amoebas $\mathcal{A}(f)$: The spine is on the one hand a deformation retraction of the amoeba but on the other hand is a tropical variety of some specific polynomial g and therefore a tropical curve. This polynomial g resp. the spine $\mathcal{S}(f)$ can at least be approximated up to an $\varepsilon > 0$ easily if one knows, which complement components of $\mathcal{A}(f)$ exist. Hence our result allows computation of the spine of amoebas of genus at most 1 of polynomials with simplex Newton polytope.