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Extending SCIP for solving MIQCPs

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Research Center MATHEON
Mathematics for key technologies

Mixed-Integer Quadratically Constrained Programming (MIQCP)

We consider optimization problems of the form

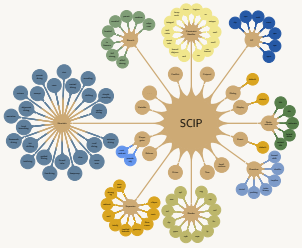
$$\begin{aligned} \text{minimize} \quad & b_0^T x + c_0 \\ \text{subject to} \quad & x^T A_j x + b_j^T x + c_j \leq 0 \quad j = 1, \ldots, m \\ & x_i \in \mathbb{Z} \quad \forall i \in I, \end{aligned}$$

where $A_j \in \mathbb{Q}^{n \times n}$, $b_j \in \mathbb{Q}^n$, $c_j \in \mathbb{Q}$, $j = 0, \ldots, m$, and $I \subseteq \{1, \ldots, n\}$.

In general, A_j does *not* need to be *positive semidefinite*.

Algorithm: LP-based branch-and-cut

We extend the *Constraint Integer Programming* framework SCIP [1] by MINLP-specific plugins.
For nonlinear constraints, we generate a suitable linear outer approximation and apply domain propagation and primal heuristics.



Currently implemented [5]

- reformulation for products with binary variables
- recognition of convex quadratic functions
- separation for convex and nonconvex quad. constraints
- domain propagation for quadratic constraints
- handling of second-order-cone (SOC) constraints
- local search heuristic: fix integers, solve *sub-QCP* locally
- interfaces to GAMS, MPS, and ZIMPL

Nonlinear RENS Heuristic

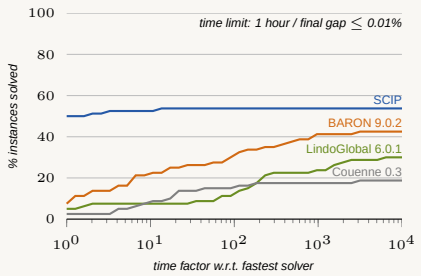
Relaxation **E**nforced **N**eighborhood **S**earch heuristic [2]:
fix integer variables that take integral value in optimal solution of LP relaxation and solve remaining *sub-MIQCP*

Undercover Heuristic [3]

construct (and solve) *sub-MIP* by fixing as few nonlinear variables as possible to their value in an optimal solution of the LP or NLP relaxation

Computational Results

Benchmark



80 MIQCP benchmark instances from MINLPLib, H. Mittelmann, J.P. Vielma

Impact of Single Components

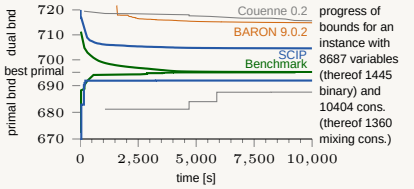
feature that is disabled	solved	primal bound	dual bound	solve time	mean time
binary var. reform					+11%
SOC upgrade					+1098%
convexity detect.					+478%
domain prop.					+7%
MIP cuts					+3%
hybrid branching					+67%
primal heuristics					+2%

each bar indicates the number of instances for which disabling a specific feature has a positive or negative effect w.r.t. the considered performance metric

Application: Mine Prod. Scheduling [6]

Stockpiles lead to *Mixing constraints*:

$$\frac{f^r_{A,t}}{f^p_{A,t}} = \frac{f^r_{B,t}}{f^p_{B,t}} = \ldots$$

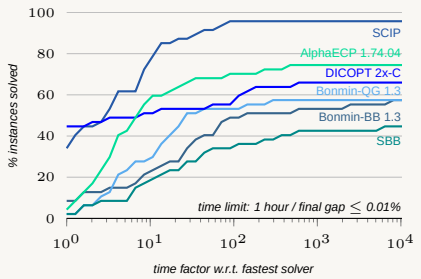


progress of bounds for an instance with 8687 variables (thereof 1445 binary) and 10404 cons. (thereof 1360 mixing cons.)

Beyond MIQCP

Convex MINLP

QG-like algorithm (gradient-based cuts)



47 convex MINLP benchmark instances from P. Bonami, M. Kilinç, J. Linderoth (2009)

Pseudo-Boolean Optimization [4]

Pseudo-Boolean Constraints:

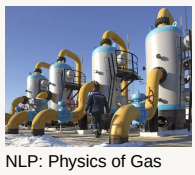
$$\sum_{j=1}^{t_i} a_{ij} \cdot \prod_{k \in I_{ij}} x_k \cdot \prod_{k \in \bar{I}_{ij}} (1 - x_k) \geq b_i$$

$$x_k \in \{0, 1\}, k \in I_{ij} \cup \bar{I}_{ij}$$

- replace multiplications by new variables and *AND-constraints*
- specialised separation and propagation for AND-constraints (small initial relax., separate strong ineq., always propagate)
- Winner in 3 out of 4 categories at Pseudo-Boolean Evaluation 2009

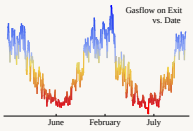
Application: Gas Transport

IP: Network Configuration



NLP: Physics of Gas

CP: Legal Requirements



SP: Uncertainties

References

[1] T. Achterberg. SCIP: Solving Constraint Integer Programs, Mathematical Programming Computation 1 (2009), pp. 1–41
[2] T. Berthold. RENS – Relaxation Enforced Neighborhood Search, ZIB-Report 07-28
[3] T. Berthold, A. Gleixner. Undercover – a primal heuristic for MINLP based on sub-MIPs generated by set covering, ZIB-Report 09-40
[4] T. Berthold, S. Heinz, M.E. Pfetsch. Nonlinear pseudo-Boolean optimization: relaxation or propagation?, Proc. of SAT 2009, pp. 441–446
[5] T. Berthold, S. Heinz, S. Vigerske. Extending a CIP framework to solve MIQCPs, ZIB-Report 09-23
[6] A. Bley, A. Gleixner, T. Koch, S. Vigerske. Comparing MIQCP solvers to a specialised algorithm for mine production scheduling, ZIB-Report 09-32

Cooperations

GAMS
SIEMENS
IBM