Adaptive discretization of convex multistage stochastic programs

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MSP[I]

$$\min_x \quad F_I(x,\xi) := \sum_{i\in I} p_i^I f(x_i,\xi_i)$$
$$\text{s.t.} \quad g_{t(n)}(x^1,\dots,x^n,\zeta^n) \le 0,\; x^n \in X_{t(n)}, \quad n \in N(I)$$

$f(x,\xi)$, $g_t(x,\xi)$ convex in x, X_t convex, $t = 1,\dots,T$

Notation

ξ_i	$i \in I$	scenarios of stoch. process ξ	$\xi^n := \xi_{i,t(n)}$ $\quad \zeta^n := (\xi^1,\dots,\xi^n)$
x_i	$i \in I$	decision vector for scenario i	$x^n := x_{i,t(n)}$
p_i^I	$i \in I$	scenario probabilities	
$N(I)$		nodes of scenario tree defined by scenarios in I	
$t(n)$		timestage of node n	T number of timestages

Task: For scenarios I with tree $N(I)$, find scenarios $J \subset I$ and probabilities p_j^J, $j \in J$, such that

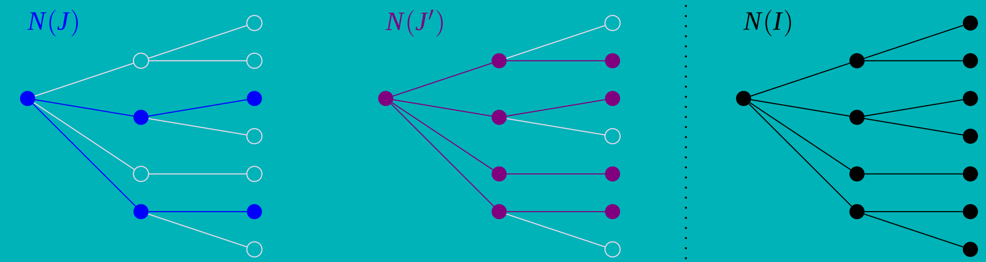
$|\mathrm{val}(\mathrm{MSP}[J]) - \mathrm{val}(\mathrm{MSP}[I])|$ and $|J|$ are small.

Consider subsets

$J \subset J' \subset I$,

and aggregations

$j: I \to J, \quad j': I \to J'$.



$$\Delta_1 := \mathrm{val}(\mathrm{MSP}[J]) - \mathrm{val}(\mathrm{MSP}[J'])$$

1.: $\mathrm{val}(\mathrm{MSP}[J]) \gg \mathrm{val}(\mathrm{MSP}[J'])$ $\quad (\Delta_1 \gg 0)$

$$\min_x \quad \sum_{j\in J} \sum_{j'\in J': j(j')=j} p_{j'}^{J'} \|x_j - x_{j'}\|$$
$$\text{s.t.} \quad g_{t(n)}(x^1,\dots,x^n,\zeta^n) \le 0,\; x^n \in X_{t(n)}, \quad n \in N(J')$$
$$F_{J'}(x,\xi) \le \mathrm{val}(\mathrm{MSP}[J']) + \varepsilon$$

$\Rightarrow$ Refine aggregation (J, j) at scenarios $j \in J$ with high $\sum_{j'\in J': j(j')=j} p_{j'}^{J'} \|x_j - x_{j'}\|$.

2.: $\mathrm{val}(\mathrm{MSP}[J']) \gg \mathrm{val}(\mathrm{MSP}[J])$ $\quad (\Delta_1 \ll 0)$

$x \in \mathrm{argmin}(\mathrm{MSP}[J])$, $x' \in \mathrm{argmin}(\mathrm{MSP}[J'])$:

$$\Delta_1 = \sum_{j\in J} \sum_{j'\in J': j(j')=j} p_{j'}^{J'} \left(f(x_{j'},\xi_{j'}) - f(x_j,\xi_j)\right)$$

$\Rightarrow$ Refine aggregation (J, j) at scenarios $j \in J$ with high $\sum_{j'\in J': j(j')=j} p_{j'}^{J'} \left(f(x_{j'},\xi_{j'}) - f(x_j,\xi_j)\right)$.

$$\Delta_2 := \mathrm{val}(\mathrm{MSP}[J']) - \mathrm{val}(\mathrm{MSP}[I])$$

Stability (Heitsch, Römisch, Strugarek '05)

$f(x,\xi)$, $g_t(x,\xi)$ linear, X_t polyhedral, $t = 1,\dots,T$:

$$|\Delta_2| \le L \left(d(J',I) + \sum_{t=2}^{T-1} D_t(\mathcal{F}_t(J'), \mathcal{F}_t(I)) \right)$$

$\mathcal{F}_t(I)$	filtration defined by scenarios I at time t
$d(I,J)$	distance of (discrete) probability measures defined by $(\xi_i, p_i^I)_{i\in I}$ and $(\xi_j, p_j^J)_{j\in J}$
$D_t(\mathcal{F}_t, \mathcal{G}_t)$	distance of filtrations (not efficiently computable)

Forward Selection (Dupačová et.al. '03)

Select scenarios $K \subseteq I \setminus J'$ and redistribute probabilities such that $|K| \le k$ and $d(J' \cup K, I) \to \min$

$T = 2$: $|\Delta_2| \le L\, d(J' \cup K, I)$

$T > 2$: open problem (Heitsch, Römisch '05)

Bibliography

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