A Fraïssé limit of nilpotent groups of finite exponent

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Abstract

Let $\mathbb{K}^P_{2,p}$ (where $2 < p$) be the class of all finite nilpotent groups of class 2 and of exponent p with an additional predicate for a subgroup of the center that contains the commutator subgroup. The Fraïssé limit D of this class exists. Non-forking is described for $\mathrm{Th}(D)$.

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1 Introduction

Let $\mathbb{G}_{2,p}$ (where $2 < p$) be the variety of the nilpotent groups of class 2 and of exponent p. Let $\mathbb{K}_{2,p}$ be the subset of finite structures in $\mathbb{G}_{2,p}$. Our aim is to construct the Fraïssé-limit of $\mathbb{K}_{2,p}$. We follow the presentation of W. Hodges ([4], Theorem 7.1.2) of Fraïssé's result [3]:

Theorem *Let L be a countable signature and let $\mathbb{K}$ be a non-empty finite or countable set of finitely generated L-structures which has HP, JEP, and AP. Then there is an L-structure D, unique up to isomorphism such that*

$$\begin{array}{ll} D & \textit{has cardinality } \leq w, \\ \mathbb{K} & \textit{is the age of } D, \textit{ and} \\ D & \textit{is ultrahomogeneous.} \end{array}$$

D is called the Fraïssé limit of $\mathbb{K}$.

The age of a structure D is the class of all finitely generated structures that can be embedded in D.

A structure D is ultrahomogeneous if every isomorphism between finitely generated substructures of D extends to an automorphism of D.

Herditary property (HP): If $A \in \mathbb{K}$ and B is a finitely generated substructure of A then B is isomorphic to some structure in $\mathbb{K}$.

Joint embedding property (JEP): If A, B are in $\mathbb{K}$, then there is C in $\mathbb{K}$ such that both A and B are embeddable in C.

Amalgamation property (AP): If A, B, C are in $\mathbb{K}$ and $e : A \to B$, $f : A \to C$ are embeddings, then there are D in $\mathbb{K}$ and embeddings $g : B \to D$ and $h : C \to D$ such that $ge = fh$.

Note that conversely the age of a Fraïssé limit has the properties HP, JEP, and AP. It is easily seen that $\mathbb{K}_{2,p}$ does not satisfy AP: Let A be an abelian p-group such that $e(A)$ is in the commutator subgroup of B and $f(A)$ is not in the center of C.
Let L^P be the group language with an additional predicate P. We can apply Fraïssé's Theorem if we replace $\mathbb{K}_{2,p}$ by the following class $\mathbb{K}^P_{2,p}$. Let $\mathbb{K}^P_{2,p}$ be the class of all L^P-structures where the L-reduct is a group in $\mathbb{K}_{2,p}$ and P describes a subgroup of the center that contains the commutator subgroup.
In the next section we show:

Theorem 1.1 *$\mathbb{K}^P_{2,p}$ has HP, JEP, and AP.*

Corollary 1.2 *The Fraïssé limit D of $\mathbb{K}^P_{2,p}$ exists. Hence every countable or finite group in $\mathbb{G}_{2,p}$ can be embedded into $D \restriction L$.*

Corollary 1.3 *For the Fraïssé limit D of $\mathbb{K}^P_{2,p}$ we have that $D' = P(D) = Z(D)$. Hence D is ultrahomogeneous in the language with an additional predicate for the center and the theory of D with this interpretation of P allows the elimination of quantifiers.*

Corollary 1.4 *The theory of D is $\aleph_0$-categorical.*

Let T be a complete theory. In [8] S. Shelah gave a definition of non-forking without assuming stability. He called T simple if every type over a set B does not fork over a subset A of B of cardinality at most that of T. So simplicity is a weaker property than stability. Recently B. Kim [6] showed symmetry and transitivity of non-forking in simple theories. Then B. Kim and A. Pillay [7] characterized simplicity and non-forking by properties like symmetry and transitivity and the independence theorem over a model. The only property lost in unstable simple theories is the boundedness of the number of non-forking extensions. A well-known example of such a theory is the theory of an algebraically closed field with a generic automorphism studied by Z. Chatzidakis and E. Hrushovski [2]. It is used by E. Hrushovski [5] for his applications of model theory in diophantine geometry.

Another well-known example of a simple theory is the random graph. But the Fraïssé limit D of $\mathbb{K}^P_{2,p}$ does not have a simple theory since it is easily seen that D contains an infinite chain $C(\{a_0, a_1, \ldots, a_n\})$ for all n of centralizers where $C(\{a_0, \ldots, a_{n+1}\})$ has infinite index in $C(\{a_0, \ldots, a_n\})$. This contradicts simplicity ([9]).

In section 3 we describe non-forking for D. Let $A \subseteq B$ be sets in the monster model $\mathbb{C}$ of Th(D). Let $\langle X \rangle$ be the subgroup generated by X. Let $\bar{c}$ be a tuple in $\mathbb{C}$. Then we show: tp$(\bar{c}/B)$ does not fork over A if and only if in the vectorspace $\mathbb{C}/Z(\mathbb{C})$ the subspaces $\langle B \rangle/Z(\mathbb{C})$ and $\langle \bar{c}A \rangle/Z(\mathbb{C})$ are linearly independent over $\langle A \rangle/Z(\mathbb{C})$ and in the vectorspace $Z(\mathbb{C})$ for every subgroup B_I with $\langle A \rangle \subseteq B_I \subseteq \langle B \rangle$ the subspaces

$\langle \bar{c}B_I \rangle \cap Z(\mathbb{C})$ and $\langle B \rangle \cap Z(\mathbb{C})$ are linearly independent over $B_I \cap Z(\mathbb{C})$. In fact we show this for non-dividing and then that non-forking is non-dividing for this theory.

This work is inspired by the construction of a new uncountably categorical group in [1]. There we only consider nilpotent groups G of class 2 and exponent p where $G' = Z(G)$. In [1] there are further essential restrictions on the groups and embeddings that are used in the amalgamation to get the final structure D. But in both papers we work in fact with bilinear maps instead of groups.

2 Amalgamation in $\mathbb{G}^P_{2,p}$

To get the desired Fraïssé limit we amalgamate finite bilinear maps. Let p be a prime greater than 2. Let G be a group. We use $[a, b] = a^{-1}b^{-1}ab$. Z or $Z(G)$ denotes the center of G and $\langle X \rangle$ the subgroup generated by the elements of the subset X of G. Then the commutator subgroup G' is $\langle \{[a, b] : a, b \in G\} \rangle$. Let $\mathbb{G}^P_{2,p}$ be the category of all L^P-structures G that satisfy

- The reduct of G to the group-language is nilpotent of class 2 and has exponent p.
- $P(G)$ is a subgroup of $Z(G)$ that contains G'.

The morphisms of $\mathbb{G}^P_{2,p}$ are the monomorphisms. Often we write only $\mathbb{G}^P$. $\mathbb{G}^P$ has HP. We show AP for $\mathbb{G}^P$. JEP follows.
If G is in $\mathbb{G}^P$, then $[\ ,\ ]$ defines an alternating bilinear map of $V \times V$ into W, where $V = G/P(G)$ and $W = P(G)$. V and W can be considered as vector spaces over $\mathbb{F}_p$, the field with p elements.
The image of $[\ ,\ ]$ generates a subspace of W. This construction of $\langle V, W, [,] \rangle$ gives us an 1-1 correspondence F between the isomorphism-types of $\mathbb{G}^P$ and $\mathbb{B}^P$, where $\mathbb{B}^P$ is the category of all alternating bilinear maps $\beta : V \times V \to W$ where V and W are $\mathbb{F}_p$-vector spaces. A morphism of $\mathbb{B}^P$ from $\langle V_1, W_1, \beta_1 \rangle$ to $\langle V_2, W_2, \beta_2 \rangle$ is a pair (f, g) of vector space monomorphisms $f : V_1 \to V_2$ and $g : W_1 \to W_2$ such that $\beta_2(f \times f) = g\beta_1$:

$$\begin{array}{ccc} V_1 \times V_1 & \xrightarrow{\beta_1} & W_1 \\ \downarrow f \quad \downarrow f & & \downarrow g \\ V_2 \times V_2 & \xrightarrow{\beta_2} & W_2 \end{array} .$$

If G, H, and f are in $\mathbb{G}^P$ where f is an embedding of G in H, then $F(f)$ is $(\overline{f}, f \downarrow P)$ where $\overline{f}$ is the embedding of $G/P(G)$ into $H/P(H)$ induced by f and $f \downarrow P$ is the restriction of f to $P(G)$. Similar as in [1] we can show:

Lemma 2.1 i) *F is a functor from $\mathbb{G}^P$ onto $\mathbb{B}^P$ that is 1-1 on the level of objects.*
ii) *In $\mathbb{G}^P$ we consider embeddings e_0 of G_0 into G and e_1 of H_0 into H. Let f_0 be an isomorphism between G_0 and H_0. Assume that there is an morphism (g, h) that embeds $F(G)$ into $F(H)$ such that*

$$
\begin{array}{ccc}
F(G_0) & \xrightarrow{F(e_0)} & F(G) \\
\Big\downarrow F(f_0) & & \Big\downarrow (g,h) \\
F(H_0) & \xrightarrow{F(e_1)} & F(H)
\end{array} .
$$

Then there is an embedding f of G into H such that $F(f) = (g, h)$ and

$$
\begin{array}{ccc}
G_0 & \xrightarrow{e_0} & G \\
\Big\downarrow f_0 & & \Big\downarrow f \\
H_0 & \xrightarrow{e_1} & H
\end{array} .
$$

Lemma 2.2 *If $\mathbb{B}^P$ has AP, then $\mathbb{G}^P$ has AP.*

Proof. Assume we have

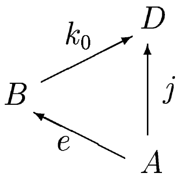
$$
\begin{array}{ccc}
B & & C \\
{\scriptstyle e}\nwarrow & & \nearrow{\scriptstyle f} \\
& A &
\end{array}
$$

in $\mathbb{G}^P$. By assumption we can amalgamate the F-images in $\mathbb{B}^P$. By Lemma 2.1i) the amalgam can be written as $F(D)$. Hence we have

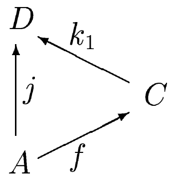
$$
\begin{array}{ccc}
& F(D) & \\
{\scriptstyle (g_0,h_0)}\nearrow & & \nwarrow{\scriptstyle (g_1,h_1)} \\
F(B) & & F(C) \\
{\scriptstyle F(e)}\nwarrow & & \nearrow{\scriptstyle F(f)} \\
& F(A) &
\end{array}
$$

By Lemma 2.1 again there is an $\mathbb{G}^P$-embedding j of A into D such that $F(j) = (g_0, h_0)F(e) = (g_1, h_1)F(f)$. Let us assume that $G_0 = H_0$ in Lemma 2.1ii). We apply Lemma 2.1ii) to the situation $A = G_0 = H_0$, $B = G$, $D = H$, $e = e_0$, $j = e_1$, and

$(g_0, h_0) = (g, h)$.
Then we obtain an embedding k_0 of B into D such that $F(k_0) = (g_0, h_0)$ and

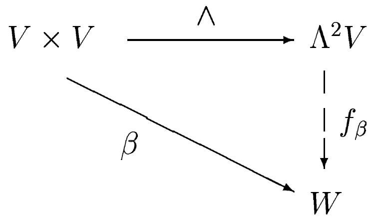


Analogously we have k_1 with $F(k_1) = (g_1, h_1)$ and



□

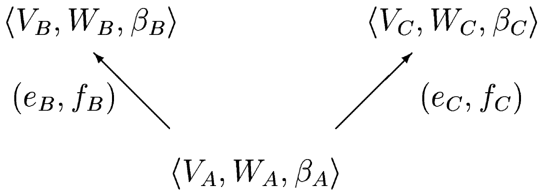
Note the following:
For every vector space V there is a free alternating bilinear map $\Lambda : V \times V \to \Lambda^2 V$ which is defined by the following property: For every alternating bilinear map $\beta : V \times V \to W$ from $I\!B^P$ there is a unique linear map $f_\beta : \Lambda^2 V \to W$ such that



$\Lambda^2 V$ is called the exterior square of V. β is completely determined by f_β.
It remains to amalgamate finite maps in $I\!B^P$.

Lemma 2.3 *$I\!B^P$ has AP.*

Proof. We consider



Let V_D be the vectorspace amalgam $V_B \bigoplus_{V_A} V_C$ with the corresponding embeddings $g_B : V_B \to V_D$ and $g_C : V_C \to V_D$. Let $b_1 \ldots b_n a_1 \ldots a_m c_1 \ldots c_n$ be a basis of the vectorspace V_D with $a_i \in g_B e_B(V_A) = g_C e_C(V_A)$, $b_i \in g_B(V_B)$, $c_i \in g_C(V_C)$, $b_1, \ldots, b_n$ are linearly independent over $g_B e_B(V_A)$ and $c_1, \ldots, c_n$ linearly independent over $g_B e_B(V_A)$.

Let W_D be $W_B \bigoplus_{W_A} W_C \oplus \bigoplus_{\substack{1 \le i \le m \\ 1 \le j \le k}} \langle b_i \wedge c_j \rangle$ where the $b_i \wedge c_j$ are new elements linearly independent over $W_B \bigoplus_{W_A} W_C$. We have canonical embeddings $h_B : W_B \to W_D$ and $h_C : W_C \to W_D$. Finally we define β_D to be β_B on V_B, β_C on V_C, and $\beta_D(b_i, c_j) = b_i \wedge c_j$. We see that $\langle V_D, W_D, \beta_D \rangle$ is well-defined and has the desired properties. □

The bilinear map $\langle V_D, W_D, \beta_D \rangle$ constructed in the proof of Lemma 2.3 is called the free amalgam of $\langle V_B, W_B, \beta_B \rangle$ and $\langle V_C, W_C, \beta_C \rangle$ over $\langle V_A, W_A, \beta_A \rangle$. The corresponding group is called the free amalgam of B and C over A.

3 Non-forking

We work in a big saturated model $\mathbb{C}$ of a complete theory T. We use Z to denote the center of $\mathbb{C}$. If $p \in S(B)$ (p is a complete type over B) and f is an automorphism of $\mathbb{C}$, then $f(p) = \{\varphi(\bar{x}, f(\bar{a})) : \varphi(\bar{x}, \bar{a}) \in p\}$. Let $\mathrm{Aut}_A(\mathbb{C})$ be the set of automorphisms of $\mathbb{C}$ that fix A pointwise. S. Shelah defined ([8]):

Definition

i) Let p be in $S(B)$ and $A \subseteq B$. p divides over A, if there are automorphisms f_i in $\mathrm{Aut}_A(\mathbb{C})$ $(i < w)$ such that $\{f_i(B) : i < w\}$ is indiscernible over A and $\bigcup_{i<w} f_i(p)$ is inconsistent.

ii) p forks over A, if for some $C \supseteq B$ every extension of p over C divides over A.

iii) T is simple, if the following is true:
(Local Character) For every $p \in S(B)$ there is some $A \subseteq B$ such that p does not fork over A and $|A| \le |T|$.

We want to describe non-forking for the Fraïssé limit D of $\mathbb{K}^P_{2,p}$. Let T be $\mathrm{Th}(D)$.

Theorem 3.1 *Let $A \subseteq B$ be subsets of $\mathbb{C}$ and $\bar{c}$ be a tuple in $\mathbb{C}$.*

1) $\mathrm{tp}(\bar{c}/B)$ *does not divide over A if and only if*

$$\langle \bar{c} \rangle \cap \langle B \rangle = \langle \bar{c} \rangle \cap \langle A \rangle \textit{ modulo } Z,$$

and for every subgroup B_I with $\langle A \rangle \subseteq B_I \subseteq \langle B \rangle$

$$\langle \bar{c} B_I \rangle \cap \langle B \rangle \cap Z = \langle B_I \rangle \cap Z.$$

2) *Forking is dividing.*

Proof. First we show 1).

($\rightarrow$) $\langle\bar{c}\rangle \cap \langle B\rangle = \langle\bar{c}\rangle \cap \langle A\rangle$ modulo Z is clear by the definition of non-dividing. Furthermore we have $\mathrm{tp}(\bar{c}/B)$ does not divide over B_I for every $A \subseteq B_I \subseteq B$. Again by the definition

$$\langle\bar{c}B_I\rangle \cap \langle B\rangle \cap Z = \langle B_I\rangle \cap Z.$$

($\leftarrow$) Without loss of generality $B \setminus A$ is finite. Suppose f_i in $\mathrm{Aut}_A(\mathbb{C})$ $(i < w)$ are such that $\{f_i(B) : i < w\}$ is indiscernible over A. We show that $\bigcup_{i<w} f_i(\mathrm{tp}(\bar{c}/B))$ is consistent.

Without loss of generality A and B are subgroups. We choose a subgroup B_I such that:

- $A \subseteq B_I \subseteq B$.
- For $b \in B_I$ $\;f_i(b/Z) = b/Z$ for all $i < \omega$ and B_I/Z is maximal with respect to this property.
- $B_I \cap Z = \{b \in B \cap Z : f_i(b) = b \text{ for } i < w\}$.

Let $\langle V_{B_I}, W_{B_I}, \beta_{B_I}\rangle$, $\langle V_C, W_C, \beta_C\rangle$, $\langle V_B, W_B, \beta_B\rangle$, and $\langle V_E, W_E, \beta_E\rangle$ be the bilinear maps in $\mathbb{B}^P$ corresponding to B_I, $\langle\bar{c}B_I\rangle$, B, and $\langle\bar{c}B\rangle$ respectively.

Let $\langle V_F, W_F, \beta_F\rangle$ be the free amalgam of $\langle V_C, W_C, \beta_C\rangle$ and $\langle V_B, W_B, \beta_B\rangle$ over $\langle V_{B_I}, W_{B_I}, \beta_{B_I}\rangle$ as described in Lemma 2.3. W.l.o.g. we assume that all embedding are the identity. Then

$$\begin{aligned} V_F &= V_C \bigoplus_{V_{B_I}} V_B, \\ W_F &= W_C \bigoplus_{W_{B_I}} W_B \oplus \bigoplus_{\substack{i<m \\ j<n}} (c_i \wedge b_j), \end{aligned}$$

where $c_0, \ldots, c_{m-1}$ is a basis of V_C over V_{B_I}, $b_0, \ldots, b_{n-1}$ is a basis of V_B over V_{B_I}, and β_F is defined by

$$\beta_F \restriction V_C = \beta_C, \quad \beta_F \restriction V_B = \beta_B \quad \text{and} \quad \beta_F(c_i, b_j) = c_i \wedge b_j.$$

Now we compare $\langle V_E, W_E, \beta_E\rangle$ and $\langle V_F, W_F, \beta_F\rangle$. By the first condition of the theorem $V_E = V_C \bigoplus_{V_{B_I}} V_B$ and by the second condition $W_C \bigoplus_{W_{B_I}} W_B$ can be considered as a subspace of W_E in the canonical way.

Hence $\langle V_E, W_E, \beta_E\rangle$ is an amalgam of $\langle V_C, W_C, \beta_C\rangle$ and $\langle V_B, W_B, \beta_B\rangle$ over $\langle V_{B_I}, W_{B_I}, \beta_{B_I}\rangle$. We obtain $V_E = V_F$, $W_E = W_F/H$ where H is a subspace of W_F with $H \cap (W_C \bigoplus_{W_{B_I}} W_B) = \langle 0\rangle$, and β_E is induced by β_F.

Our aim is to show that $\bigcup_{i<n} f_i(\mathrm{tp}(\bar{c}/B))$ is consistent for every $n < \omega$.

Let $\langle V_U, W_U, \beta_U\rangle$ be the bilinear map that corresponds to $\langle \bigcup_{i<n} f_i(B)\rangle$. As most of the

bilinear maps before it lives in $\langle V_{\mathbb{C}}, W_{\mathbb{C}}, \beta_{\mathbb{C}}\rangle$, the bilinear map of $\mathbb{C}$. Let $\langle V_i, W_i, \beta_i\rangle$ be the bilinear map that corresponds to $f_i(B)$. $\langle V_0, W_0, \beta_0\rangle$ is $\langle V_B, W_B, \beta_B\rangle$. By indiscernibility of the $f_i(B)$ over B_I we have that the V_i are linearly independent modulo V_{B_I} and the W_i are linearly independent modulo W_{B_I}. Hence $V_U = \bigoplus_{\substack{V_{B_I}\\ i<n}} V_i$ and $\bigoplus_{\substack{W_{B_I}\\ i<n}} W_i$ is a subspace of W_U.

Since we have quantifier elimination for T $\bigcup_{i<n} f_i(\mathrm{tp}(\bar{c}/B))$ can be considered as a set of quantifier free formulas in the group language with an extra predicate for the centre. Since T is the theory of a Fraïssé limit it is sufficient to find a structure in $\mathbb{G}^P$ that is generated by $\langle \bigcup_{i<n} f_i(B)\rangle$ and a realization of $\bigcup_{i<n} f_i(\mathrm{tp}(\bar{c}/B))$. By Lemma 2.1 we can work in $\mathbb{B}^P$.

Let $\langle V_X, W_X, \beta_X\rangle$ be the free amalgam of $\langle V_C, W_C, \beta_C\rangle$ and $\langle V_U, W_U, \beta_U\rangle$ over $\langle V_{B_I}, W_{B_I}, \beta_{B_I}\rangle$. Then $\langle V_X, W_X, \beta_X\rangle$ contains the free amalgam $\langle V_{F_i}, W_{F_i}, \beta_{F_i}\rangle$ of $\langle V_C, W_C, \beta_C\rangle$ and $\langle V_i, W_i, \beta_i\rangle$ for $i < n$. It is isomorphic to $\langle V_F, W_F, \beta_F\rangle$. Each W_{F_i} contains an image H_i of H according to the canonical isomorphism between $\langle V_F, W_F, \beta_F\rangle$ and $\langle V_{F_i}, W_{F_i}, \beta_{F_i}\rangle$. Then $H_i \cap (W_C \bigoplus_{W_{B_I}} W_i) = \langle 0\rangle$, $H_i \subseteq W_{F_i}$ and the W_{F_i} are linearly independent modulo W_C. Hence

$$W_X = W_C \underset{W_{B_I}}{\oplus} \bigoplus_{\substack{W_{B_I}\\ i<n}} (W_i) \oplus \bigoplus_{i<n} H_i \oplus \bigoplus_{i<n} K_i \oplus K$$

where $W_{F_i} = W_i \oplus H_i \oplus K_i$.

Then $\langle V_H, W_H, \beta_H\rangle$ with $V_H = V_X$, $W_H = W_X/\bigoplus_{i<n} H_i$ and β_H is induced by β_X is the desired structure.

It contains $\langle V_C, W_C, \beta_C\rangle$ and $\langle V_i, W_i, \beta_i\rangle$ and these both generate a structure isomorphic to $\langle V_E, W_E, \beta_E\rangle$ where the isomorphism comes from the identity for $\langle V_C, W_C, \beta_C\rangle$ and the canonical isomorphism of $\langle V_B, W_B, \beta_B\rangle$ onto $\langle V_i, W_i, \beta_i\rangle$ that is the identity on $\langle V_{B_I}, W_{B_I}, \beta_{B_I}\rangle$.

Now we show 2).

Dividing implies forking. It remains to show that non-dividing implies non-forking. For this we use the characterization of non-dividing in the first part of the theorem. So suppose $tp(\bar{c}/B)$ does not divide over A. Let C be a set that contains B. We have to show that there is a complete type over C that extends $\mathrm{tp}(\bar{c}/B)$ and does not divide over A. W.l.o.g. we assume that A, B, C are all subgroups of $\mathbb{C}$. Let $C^* \in \mathbb{G}^p_{2,p}$ be isomorphic to C such that $P(C^*)$ corresponds to $Z(\mathbb{C}) \cap C$. Similarly we use $B^* \supseteq A^*$ to denote images of B and A in $\mathbb{G}^p_{2,p}$ where $P(B^*)$ is given by the image of $B \cap Z(\mathbb{C})$. Let $\langle \bar{c}^* B^*\rangle \in G^p_{2,p}$ be an isomorphic image of $\langle \bar{c}B\rangle$ extending the isomorphism between B and B^*. Again P is given by $Z(\mathbb{C})$. Let E^* be the free amalgam of $\langle \bar{c}^* B^*\rangle$ and C^* over B^*. Then $\bar{c}^*$, A^* and C^* fulfil the condition of 1). Now we extend the isomorphism of C^* onto C to an embedding of E^* into $\mathbb{C}$. Let $\bar{e}$ be the image of $\bar{c}^*$ in $\mathbb{C}$. Hence $\mathrm{tp}(\bar{e}/C)$ and A fulfil the condition in 1). Therefore $\mathrm{tp}(\bar{e}/C)$ does not divide over A and it extends p, as desired. □

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