Stochastic Processes II Summer term 2008
(Stochastische Analysis)
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Exercises, 16th April

1.1 (4 points) Let $g : [a,b] \to \mathbb{R}$ be a continuous function of finite variation and let $f[a,b] \to \mathbb{R}$ be a simple function, i.e.

$$f = \sum_{i=0}^{N-1} \alpha_i \mathbb{1}_{(a_i, a_{i+1}]}$$

with $\alpha_i \in \mathbb{R}, N \in \mathbb{N}$ and a fixed partition $a \leq a_1 < \ldots < a_N \leq b$. Compute the Riemann-Stieltjes integrals $\int_a^b f(x)dg(x)$ and $\int_a^b g(x)df(x)$.

1.2 (4 points) Let $g \in C^1([a,b])$ and $f \in C([a,b])$. Prove that

$$\int_a^b f(x)dg(x) = \int_a^b f(x)g'(x)dx.$$

1.3 (5 points) We call a function $F : [0,1] \to \mathbb{R}$ a "classical" integrator, if it has the following property: For each $h \in C[0,1]$ and each sequence of partitions $\tau^n = \{0 = t_0^n < \ldots < t_{k_n}^n = 1\}$ $(n = 1, 2, \ldots)$ such that $\|\tau^n\| := \max_{1 \leq i \leq k_n} |t_i^n - t_{i-1}^n| \to 0$ with $n \to \infty$ the sequence of Riemann-sums

$$\sum_{i=1}^{k_n} h(t_{i-1}^n) \left(F(t_i^n) - F(t_{i-1}^n) \right)$$

converges to a finite limit in $\mathbb{R}$ with $n \to \infty$. According to the tutorial any function of finite variation is a "classical" integrator. Show that also the converse holds: If F is a "classical" integrator, then it is of finite variation.
Hint: Use the Banach-Steinhaus Theorem: Let X be a Banach space, and let Y be a normed linear space. Let $(T_a)_{a \in A}$ be a family of bounded linear operators from X into Y. If for each $x \in X$ the set $(T_a(x))_{a \in A}$ is bounded, then the set $(\|T_a\|)_{a \in A}$ is bounded.

The problems 1.1 -1.3 should be solved at home and delivered at Wednesday, the 23rd April, before the beginning of the tutorial.