Back to BEE

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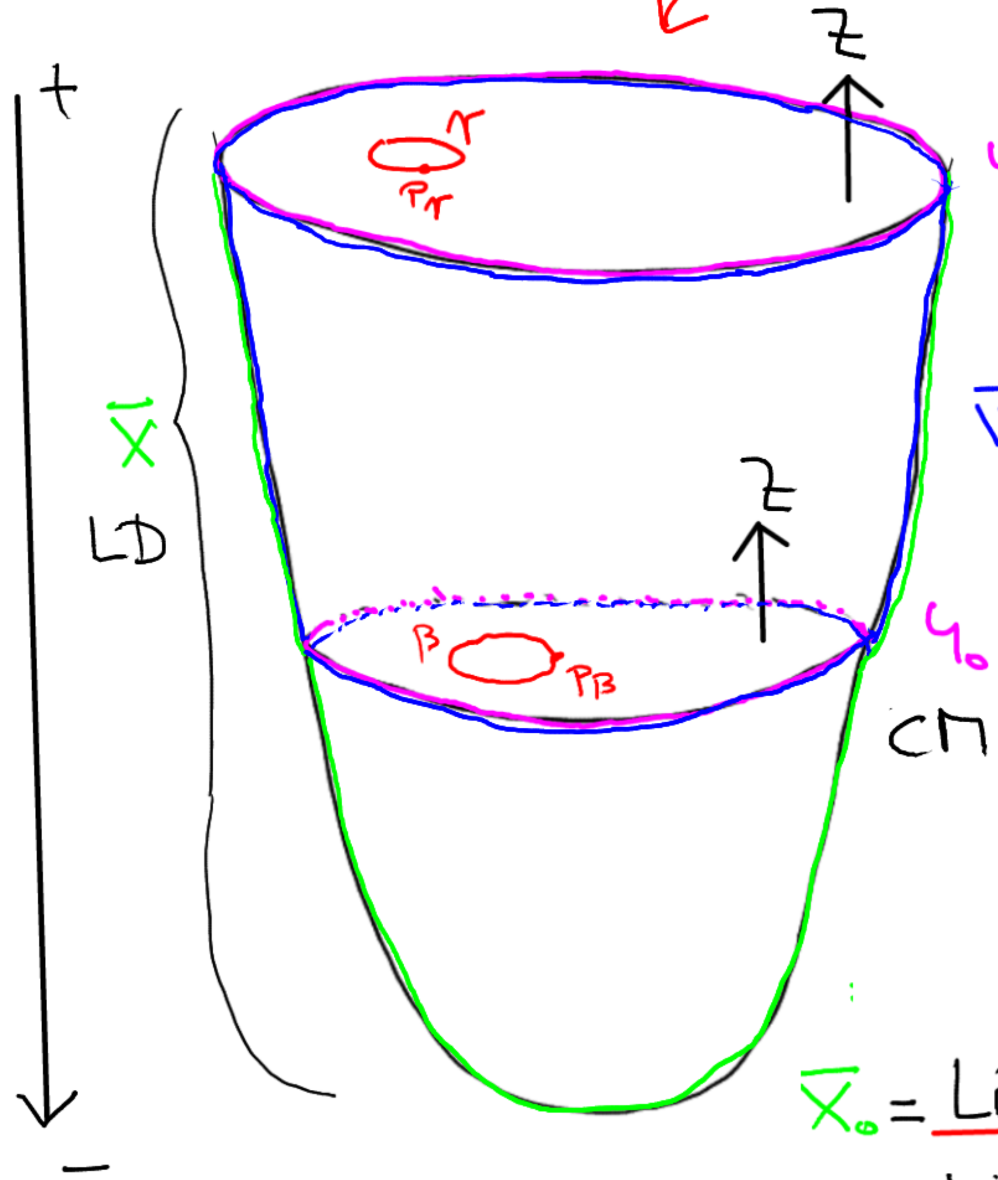
Structure

1. Geometry of BEE
2. Invariants from holomorphic curves (motivation)
3. Linearized contact homology
   - A.: counting anchored cylinders
   - B.: linearization using augmentation in tree-level DGA framework
4. Reduced and full symplectic homology in Morse-Bott
5. Outlook on BEE

1. Geometry of BEE

! Dimensionally incorrect picture !



Y = **contact mfd:**

$\alpha = \lambda|_Y$

$\overline{W}$ = **Liouville cobordism:**

cpt, $\partial^+\overline{W} = Y$, $\partial^-\overline{W} = Y_0$, $\lambda \in \Omega^1(\overline{W})$ s.t. $d\lambda$ ndeg. (Liouville form), $Z \in \mathfrak{X}(\overline{W})$ s.t. $\iota_Z d\lambda = \lambda$, $Z \pitchfork \partial\overline{W}$ outward pointing at $\partial^+\overline{W}$ and inward at $\partial^-\overline{W}$ (Liouville v.f.)

$\overline{X}_0$ = **Liouville domain:**

Liouv. cobord with $\partial^-\overline{X}_0 = \emptyset$

dim = $2n$

(a) γ : unparametrized nondeg. Reeb orbit : $[\gamma : S^1 \to Y]$ $\circlearrowleft$ dif.

multiplicity $\kappa(\gamma)$

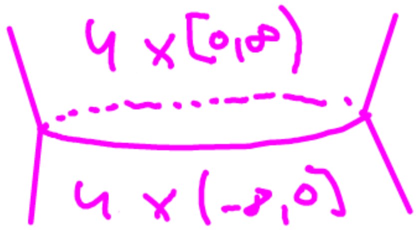
$P_\gamma \in \Gamma = \gamma(S^1)$: fixed point $\Rightarrow$ $\gamma : [0, T_\gamma] \to Y$ running along R $\kappa(\gamma)$-times

P_{good}, P_{bad}, P (bad $:\Leftrightarrow$ $\gamma = \eta^{2k}$ & $\mu(\gamma) \not\equiv \mu(\eta) \bmod 2$)

↖ simple

(b) Use flow of Z to identify collar nbhds of $+/-$ boundaries of $\overline{X}_0, \overline{X}, \overline{W}$ with $+/-$ ends of symplectizations

$Y_0 \times \mathbb{R}$, $Y \times \mathbb{R}$:

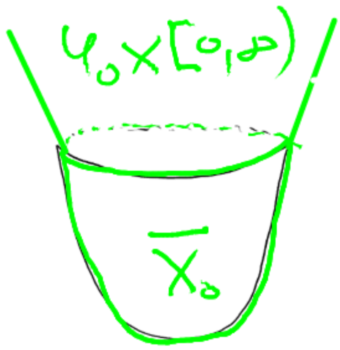


$\omega = d(e^s \alpha)$

to construct completions :

$X_0 = \overline{X}_0 \cup_{Y_0} [0,\infty) \times Y_0$:

$X = \overline{X} \cup_{Y} [0,\infty) \times Y$

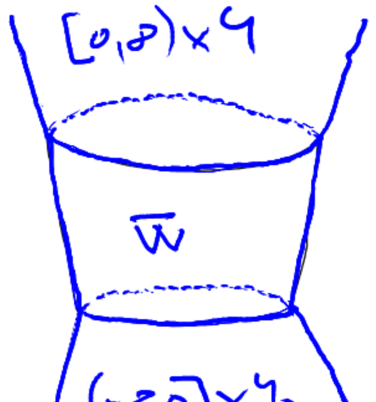


$Z \sim \frac{\partial}{\partial s}$

$W = (-\infty,0] \times Y_0 \cup_{Y_0} \overline{W}$:

$\cup_Y [0,\infty) \times Y$

Z gradient like



(c) $\overline{W}$ Weinstein : Liouville +

1 $\exists\, H : \overline{W} \to \mathbb{R}$ Morse s.t. $dH(Z) \geq \delta |Z|^2$ for $\delta > 0$ and g

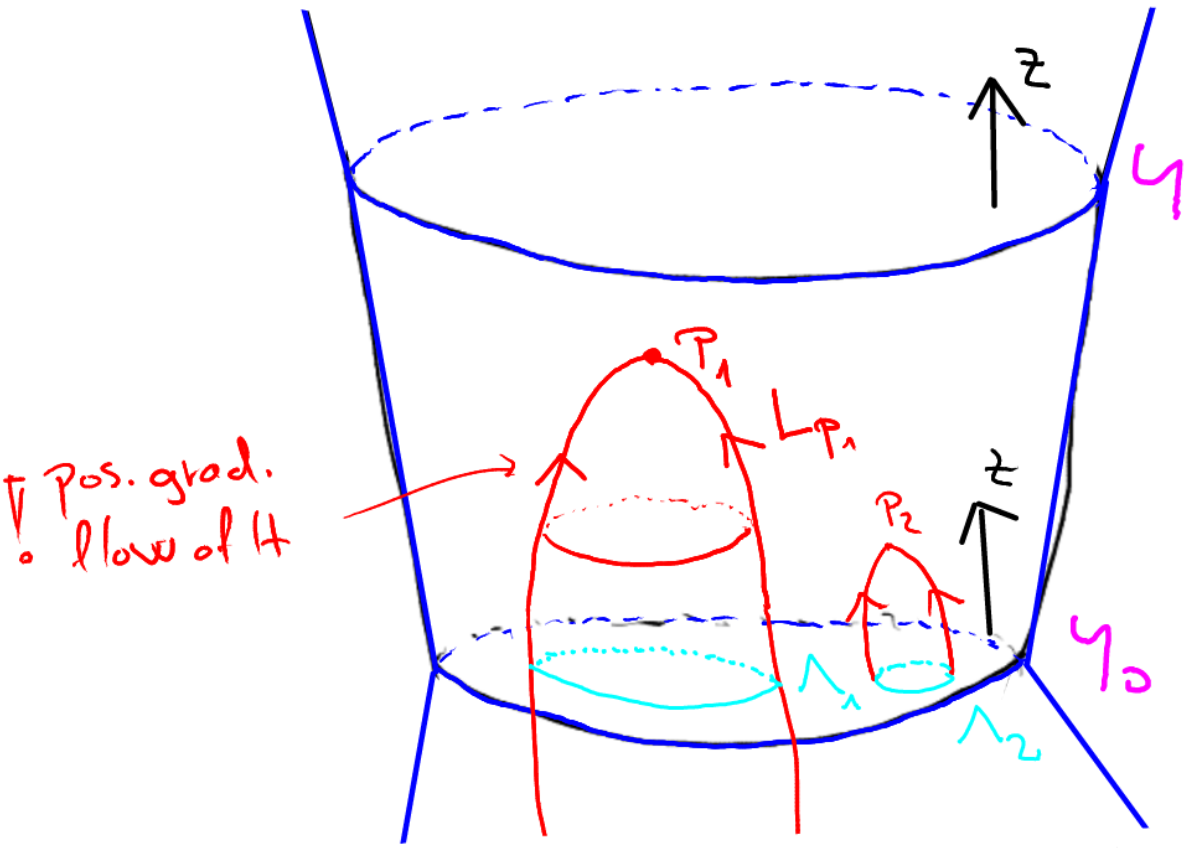
and 2 $H|_{\partial_\pm \overline{W}} = \text{const}$ and 3 $\mathrm{Crit}(H) \cap \partial \overline{W} = \phi$

Lyapunov fn. for Z

$\Longrightarrow$ assume $H|_{\partial_- W} = 0$ and extend to W s.t. $H(s) = b e^s$ on $Y \times [0,\infty)$ and $\mathrm{Crit}(H) \cap Y_0 \times (-\infty, 0] = \phi$

$p \in \mathrm{Crit}(H)$: $L_p := W^s(p)$ stable mfd wrt. Z

($\dim L_p = \mathrm{ind}(p)$, L_p isotropic, $\mathrm{ind}\,p \leq n$)



$\mathrm{ind}\,p_i = n \implies L_{p_i} \subset W$ Lagrangian $(\simeq \mathbb{R}^n)$

$\Lambda_i \subset Y_0$ Legendrian $(\simeq S^{n-1})$

"X obtained from X_0 by attaching critical Weinstein handles $D^n \times D^n$ with $\partial D^n \times D^n$ along Λ_i"

(Y obtained from Y_0 by Legendrian surgery along Λ_i)

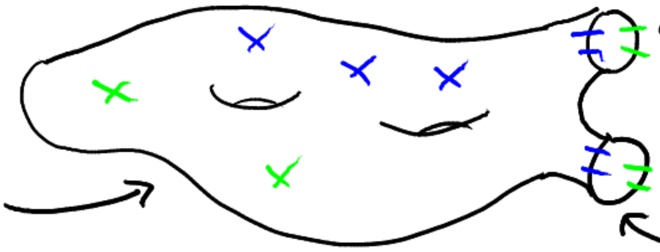
!: Proper picture $n = 1$:

(collar nbhd of $\partial \overline{W}$ $\simeq (-\infty, 0] \times Y_0 \subset X_0$)

2. Invariants from holomorphic curves (motivation)

$\overline{\mathcal{M}} = \{ \overline{\mathcal{M}}(g, \mathrm{IN}, \mathrm{out}) \}$ compactified moduli spaces of holocurves

"closed punctures" asymp. to Reeb orbits



„open punctures" asymp. to Reeb chords

boundaries lying in a Lagrangian

master equation: $\partial \overline{\mathcal{M}} + \frac{1}{2}[\overline{\mathcal{M}}, \overline{\mathcal{M}}] = 0$

from codim 1 bdd structure

breaking   sewing

(twisting morphism in suitable setting?)

$\Longrightarrow$ rich alg. structure on the v.s. gen. by Reeb orbits / chords

(most general: "quantum open-closed")

I consider the subtheories (closed under sewing / breaking)

$\overline{\mathcal{M}}^X_{cl}(0,1,1)$: $CH(X)$, $SH^+(X)$, $SH(X)$ ← full sympl.

lin. contact hom.   reduced sympl.

$\overline{\mathcal{M}}^W_{cl}(0,1,1)$: $F^W_{CH} : CH(X) \to CH(X_0)$

$F^W_{SH^+} : SH^+(X) \to SH^+(X)$

$F^W_{SH} : SH(X) \to SH(X)$

$\{ \overline{\mathcal{M}}^Y_{cl}(0,1,k) \}$: to explain linearization as twist with augmentation in the DGA setting

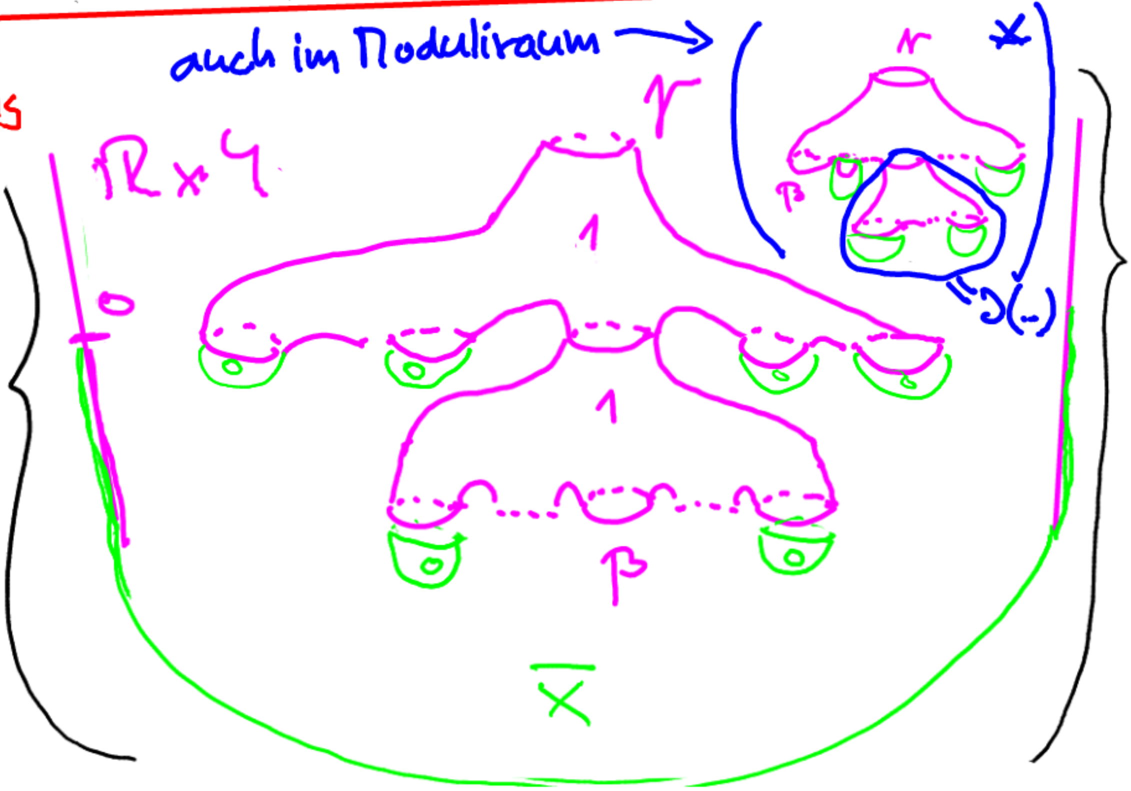
$\overline{\mathcal{M}}^X_{cl}(0,1,0)$

3. Linearized contact homology

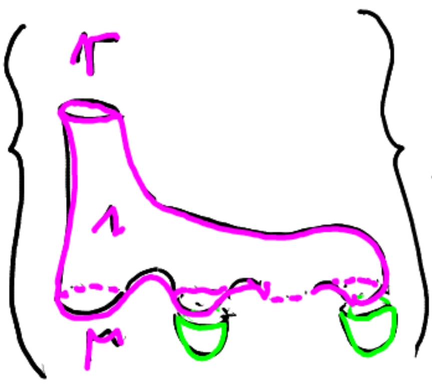
A. Using cylindrical part of master equation

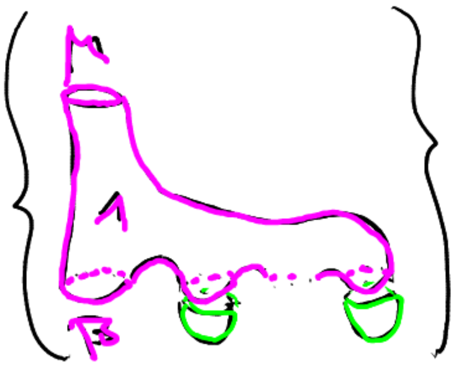
⚠ I am not drawing trivial cylinders

auch im Moduliraum →

$\partial \overline{\mathcal{M}}^X_2(\gamma;\beta) =$ 

take ∇. formally

$= \frac{1}{2}$ 

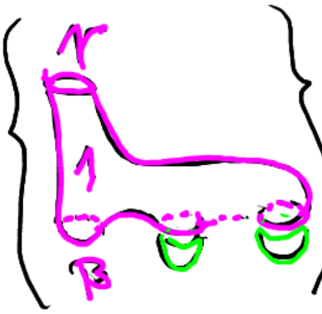
$\times$ 

(Einstein sum. convention)

$+ (*) = 0$

⚠ in BEE just γ, not q^γ

$CH(X) := \operatorname{span}_{\Bbbk}\{ q^\gamma \mid \gamma \in \mathcal{P}_{\text{good}}(Y),\ |q^\gamma| = \mu_{CZ}(\gamma) \}$

$d_{CH}\, q^\gamma :=$ 

$q^\beta = \displaystyle\sum_{|\beta|=|\gamma|-1} \frac{n_{\gamma\beta}}{\kappa(\beta)}\, q^\beta$

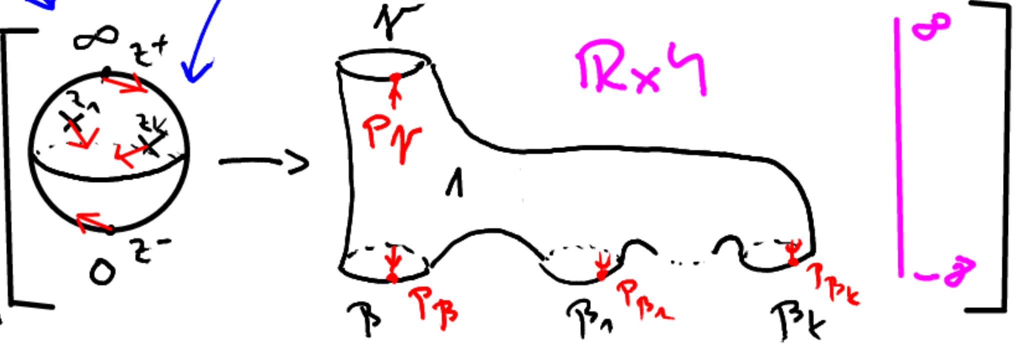
← what is it?

$d_{CH}^2 = 0$

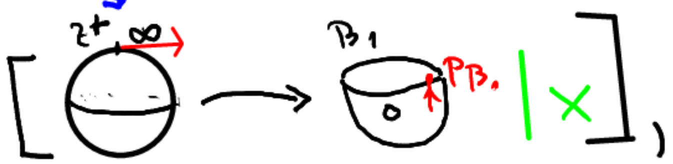
$\Longrightarrow$ $CH(X) := H(CH(X), d_{CH})$ linearized contact homology asymptotic markers free to choose

Fix cplx mfd S^2 (Recall $Aut(S^2)$ = Möbius transf determined by 3 points)

$n_{\gamma\beta} = \#$ {



Fix: +−$\mathbb{R}$ axis under $\simeq \mathbb{C}$



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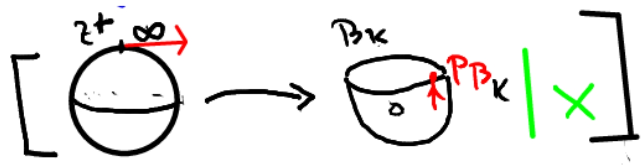


$\Big| \; \beta_i \in \mathcal{P}_{good}(Y), \; k \geq 0$ }

counting 1-dim components ($\mathbb{R}$-action)

Rem 3.2 (BEE):

"planes in X"

$$= \sum_{\substack{m \geq 0 \\ k_i \geq 0 \\ \beta_i \in \mathcal{P}_{good}(Y)}} n^+ \frac{1}{k_1! \cdots\cdots k_m!} \left(\frac{n_1^-}{\kappa(\beta_1)}\right)^{k_1} \cdots \left(\frac{n_m^-}{\kappa(\beta_m)}\right)^{k_m}$$

"spheres in $\mathbb{R}\times Y$"

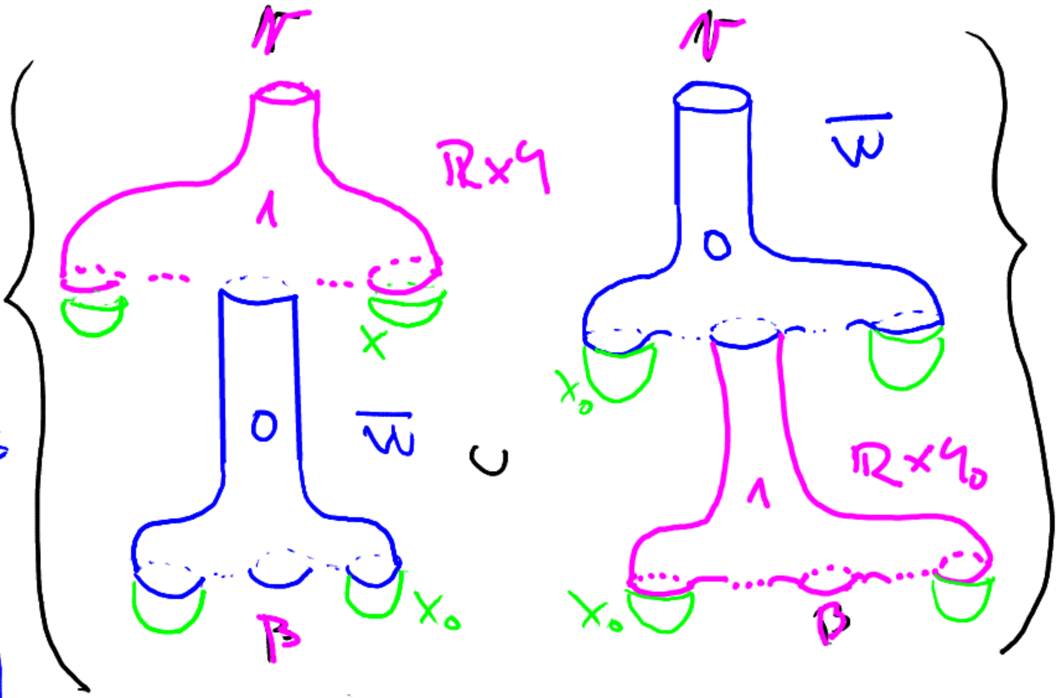
k_i punctures asympto β_i !

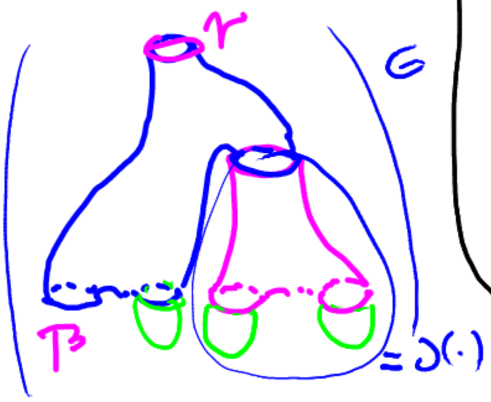
The moduli space above $=: M^Y(\gamma;\beta)$ ← $\dim |\gamma| - |\beta|$

Elements thereof $=:$ cylinders in $\mathbb{R}\times Y$ anchored in X

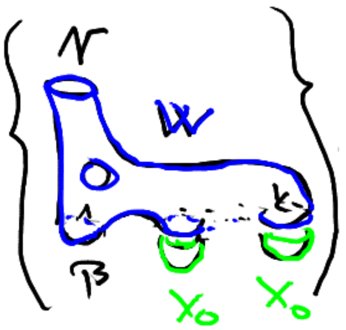
Cobordism $\overline{W}$ induces chain maps :

$\partial \mathcal{M}^{\overline{W}}_1(\gamma;\beta) =$





$\Longrightarrow \quad F^W_{CH}(q^\gamma) =$



$$q^\beta = \sum_{|\beta|=|\gamma|} \frac{n_{\gamma\beta}}{\kappa(\beta)} q^\beta$$

similar definition as that of $\mathcal{M}^Y(\gamma;\beta)$

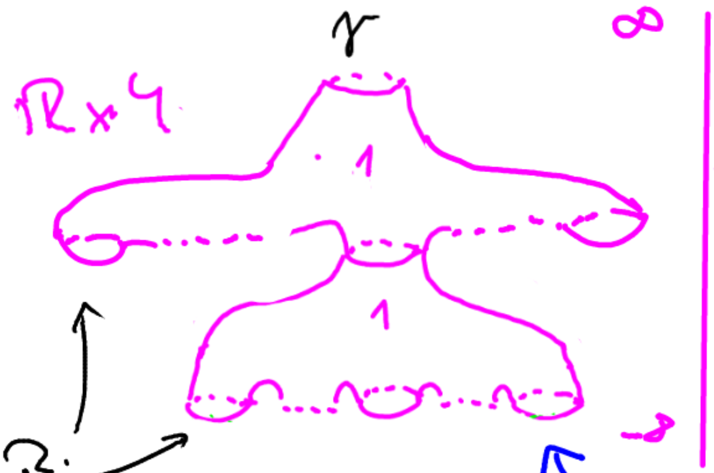
chain map : $d^{Y_0}_{CH} F^W_{CH} = F^W_{CH} d^Y_{CH}$

$n_{\gamma\beta} = \mathcal{M}^W(\gamma;\beta)$ = cylinders in W anchored in X_0 asymptotic to γ at $+$ and β at $-$

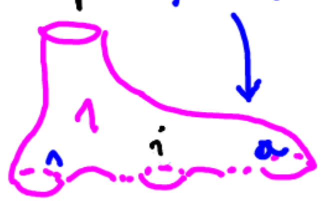
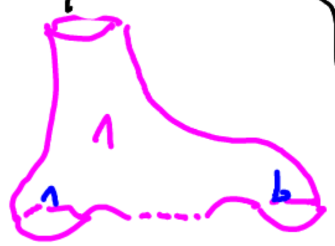
dim $|\gamma| - |\beta|$

$\Longrightarrow \quad F^W_{CH} : CH(X) \longrightarrow CH(X_0)$ cobordism induces maps of homology

B. Using tree level and disc part of master equation

$$\overline{\mathcal{M}}_2^{q}(\gamma;\beta_1,\dots,\beta_k) = \Big\{$$  $$\Big\}$$

can be ()

$$= \frac{1}{2}\sum_{\substack{a,b\geq 0\\ a+b=k+1\\ \sigma\in S_{k+1},\ i=1,\dots,a}} \frac{1}{a!}\frac{1}{b!}\Big\{$$  $$\Big\}\Big\{$$  $$\Big\}$$

$$\Rightarrow \underline{Dq^{\gamma}} := \sum_{|\beta_1|+\dots+|\beta_k|=|\gamma|-k} \frac{1}{k!}\ \#_{1\text{-dim}}\Big\{$$  $$\Big\} q^{\beta_1}\cdots q^{\beta_k} \in S(CH[1])$$

free graded sym.alg.

$$(CH[1])^i = CH^{i-1}$$

$$=$$  $$1 +$$  $$q^{\beta_1} + \frac{1}{2!}$$  $$q^{\beta_1}q^{\beta_2} + \dots$$

dim $|\gamma|-|\beta_1|-\dots-|\beta_k|-k$

$$= \left(D_{10} + D_{11} + D_{12} + \dots\right) q^{\gamma}$$

do not exist by maximum principle

extend by Leibniz rule to a derivation

$$\widehat{D} = \widehat{D_{10}} + \widehat{D_{11}} + \widehat{D_{12}} + \dots : E(CH) \longrightarrow E(CH), \qquad E(CH) := S(CH[1])$$

$\underline{|\widehat{D}| = -1}$

$$\Rightarrow \underline{\widehat{D}^2 = 0,\ \widehat{D}(1) = 0} \quad \Big(\text{morally counts}$$  $$1 +$$  $$q^{\beta} + \dots\Big)$$

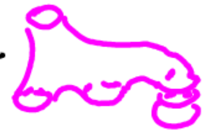
$i_j : E_j \hookrightarrow E,\ \pi_j : E \to E_j$

$$\widehat{D}^2 = 0 \iff \forall l \geq 0 : \pi_l \circ \widehat{D}^2 \circ i_1 = \sum D_{1b} \circ_i D_{1a} = 0$$

$\widehat{D}(1) = 0$ Relation $(1l)$: $a+b = l+1$, $a,b \geq 0$ connect at the i-th position

(10): $D_{10} \circ D_{11} =$  $= 0$ ← Relations from $\partial \overline{\mathcal{M}}_2^Y$

(11): $D_{11}^2 + D_{10} \circ_1 D_{20} =$  $+$  $= 0 \implies \underline{D_{11}^2 \neq 0}$ in general

(12): $D_{12} \circ D_{11} + D_{11} \circ_1 D_{12} + D_{10} \circ_1 D_{13} =$  $+$  $+$  $= 0$

not chain map, in general

* **Given a filling $\overline{X}$ of Y:**

$\partial \overline{\mathcal{M}}^X(r) = \{$  $\} = \frac{1}{2}[\,\cdot\,,\,\cdot\,]$

can not just take square for the sewing bracket

algebraically: $\beta : E(CH) \to E(0) = \Bbbk \subset E(CH)$

$\beta(q^r) := \#\{$  $\} 1$ 0-dim

$f_1 * f_2 = \mu(f_1 \otimes f_2)\delta$ convolution product

$$e^\beta = \sum_{k=0}^{\infty} \frac{1}{k!} \beta^{*k} : E(CH) \to E(CH)$$

ass. bialg.:
$\mu(v_1..v_k \otimes w_1..w_l) = v_1..v_k w_1..w_l$
$\delta(v_1..v_k) = \sum \frac{1}{k_1!} \frac{1}{k_2!}$
$\sigma \in S_k$, $k_1 + k_2 = k$
$v_{\sigma_1} \cdots v_{\sigma_{k_1}} \otimes v_{\sigma_{k_1+1}} \cdots v_{\sigma_{k_1+k_2}}$

$$e^\beta(q^{r_1} \ldots q^{r_k}) = \beta(q^{r_1}) \ldots \beta(q^{r_k}) 1$$

in general, if β has ∞-many terms  filtration and completion needed

$\beta : E(CH) \to E(0)$ $\underline{\text{augmentation}} :\iff e^\beta \circ \widehat{D} = 0$

degree 0

$$\underline{\Phi_\beta := e^\beta * 1\!\!1} : E(CH) \longrightarrow E(CH)$$

$$\Phi_\beta(q^{\gamma_1}\cdots q^{\gamma_k}) = \sum_{\substack{\sigma\in S_k\\ 0\le i\le k}} \frac{1}{i!}\frac{1}{(k-i)!}\,\beta(q^{\gamma_{\sigma_1}})\cdots\beta(q^{\gamma_{\sigma_i}})\,q^{\gamma_{\sigma_{i+1}}}\cdots q^{\gamma_{\sigma_k}}$$

- $$\Phi_{-\beta}\circ\Phi_\beta \sim \underbrace{e^{-\beta} * e^{\beta}}_{=1} * 1\!\!1 = 1\!\!1$$

$$\left(\text{using } \delta\mu = (\mu\otimes\mu)(1\!\!1\otimes J\otimes 1\!\!1)(\delta\otimes\delta)\right)$$

- $$\underline{\widehat{D}_\beta := \Phi_\beta\circ\widehat{D}\circ\Phi_{-\beta}} : E(CH) \longrightarrow E(CH)$$

$$\Longrightarrow \widehat{D}_\beta^{\,2} = 0, \quad \widehat{D}_\beta = \widehat{D_{\beta 10}} + \widehat{D_{\beta 11}} + \widehat{D_{\beta 12}} + \dots$$

computation

$$D_{\beta 1} := \pi_\ell \circ \widehat{D}_\beta \circ i_1$$

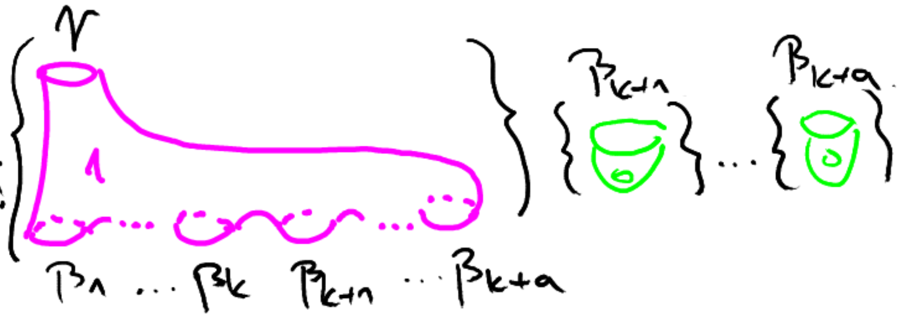
For general β, completions needed, then filtered DGA

$$\underline{\text{new DGA } (\widehat{D}_\beta, E(CH))}$$

- $$\pi_0\circ\widehat{D}^\beta = \pi_0\circ\Phi_\beta\circ\widehat{D}\circ\Phi_{-\beta} = \underbrace{e^\beta\circ\widehat{D}}_{0}\circ\Phi_{-\beta} = 0$$

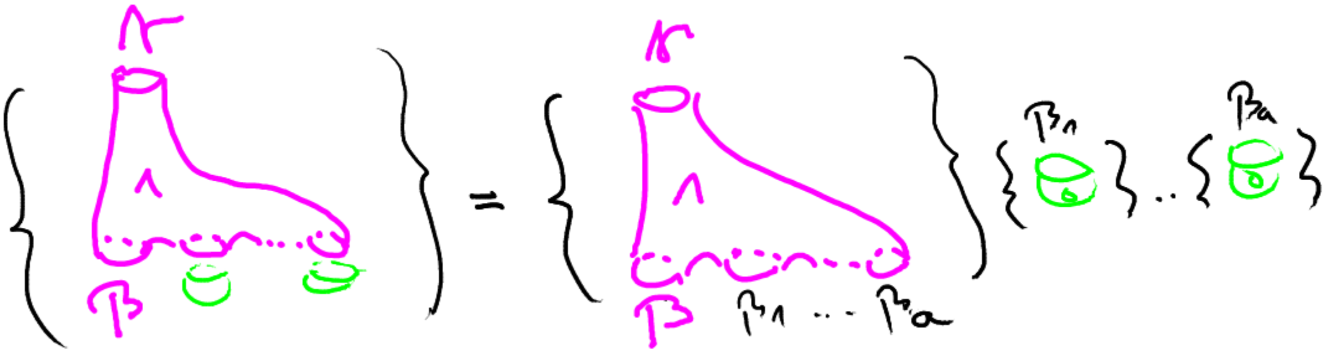
$$\Longrightarrow D_{\beta 10} = 0, \text{ and hence } \underline{D_{\beta 11}^{\,2} = 0}$$

$$D_{\beta 1 k}\, q^{\gamma} = \sum_{k,a} \frac{1}{k!}\frac{1}{a!}$$



$$q^{\beta_1} \cdots q^{\beta_k}$$

in particular, one see that $\underline{d_{CH} = D_{\beta 1 1}}$:



(up to combinatorics with multiple
covers and many punctures asymptotic to one orbit)

4. (Reduced) Symplectic Homology

reduced = positive

SH^+ ... "non-equivariant version of CH"

$$\underline{SH^+(X)} = \overset{\vee}{CH}(X) \oplus \widehat{CH}(X) \cong \overset{\vee}{CH}(X)[1]$$

i.e., $\widehat{CH}^i = \overset{\vee}{CH}^{i-1}$

$\overset{\vee}{CH}(X) \xrightarrow{\delta_{SH^+}} \widehat{CH}(X)$, $\quad \widehat{CH}(X) \xrightarrow{d_\Pi} \overset{\vee}{CH}(X)$

$\overset{\vee}{CH}(X) = \operatorname{span}_{\mathbb{k}}(\mathcal{P}(\lambda))$

recall $CH = \operatorname{span}_{\mathbb{k}}\{P_{good}\}$

$$\underline{d_{SH^+}} = \begin{pmatrix} d_{\overset{\vee}{CH}} & d_\Pi \\ \delta_{SH^+} & d_{\widehat{CH}} \end{pmatrix}$$

$d_\Pi \hat{\gamma} = \pm 2 \check{\gamma}$ if γ bad

otherwise 0

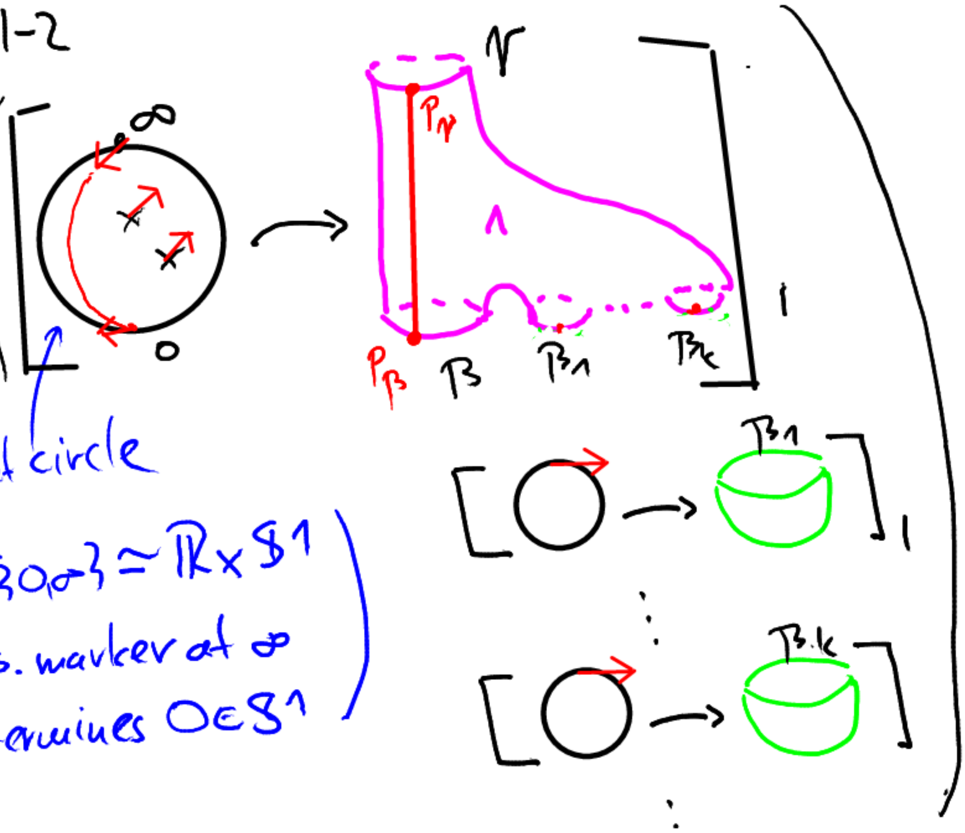
$d_{\overset{\vee}{CH}} \check{\gamma} = \sum n_{\gamma\beta} \frac{\check{\beta}}{\kappa(\beta)}$ $\qquad d_{\widehat{CH}} \hat{\gamma} = \sum n_{\gamma\beta} \frac{\hat{\beta}}{\kappa(\gamma)}$

$\underline{\dim \ |\gamma| - |\beta| - 1}$

$\underline{\overset{\vee}{\mathcal{M}}{}^{\mathcal{H}}(\gamma;\beta)}$

$$\delta_{SH^+} \check{\gamma} = \sum_{|\beta| = |\gamma| - 2} m_{\gamma\beta} \, q^{\hat{\beta}}$$

$m_{\gamma\beta} = \underset{1\dim}{\#} \Big\{ \Big(\Big[$

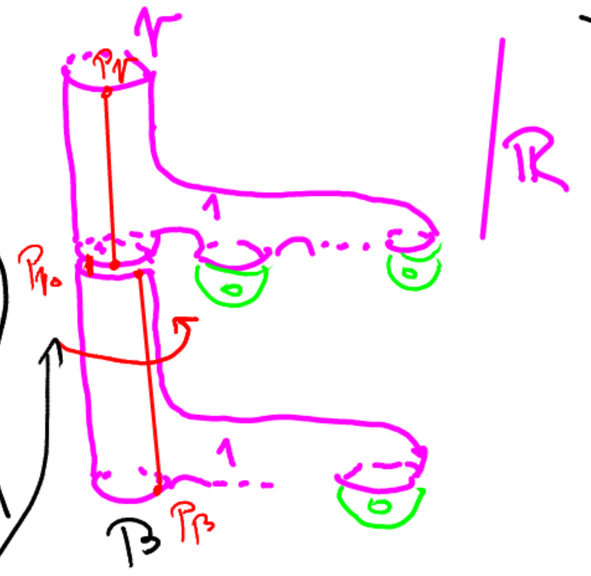


marker at 0 determined by marker at ∞ $\overset{?}{\Leftrightarrow}$

great circle

$\Big(S^2 \setminus \{0, \infty\} \cong \mathbb{R} \times S^1$ ass. marker at ∞ determines $0 \in S^1 \Big)$

$+\#$
1-dim



$$=: \widehat{\mathcal{M}}''^{4}(\gamma;\beta)$$

require cyclic order $p_{\gamma_0}, ev_+(\ell_+), ev_-(\ell_-)$

Hence, $m_{\gamma\beta}$ counts $\check{\mathcal{M}}^4(\gamma;\beta) := \check{\mathcal{M}}'^4(\gamma;\beta) \cup \check{\mathcal{M}}''^4(\gamma;\beta)$

$d^2_{SH_+} = 0 \implies SH^+(X) := H(SH^+(X), d_{SH^+})$

reduced symplectic homology

$SH(X) := SH^+(X) \oplus \mathrm{Morse}(-H)[-n]$

convention
$(V[k])^i = V^{i-k}$
$\mu(x) = \mathrm{ind}_x(-H) - n$

δ_{SH}

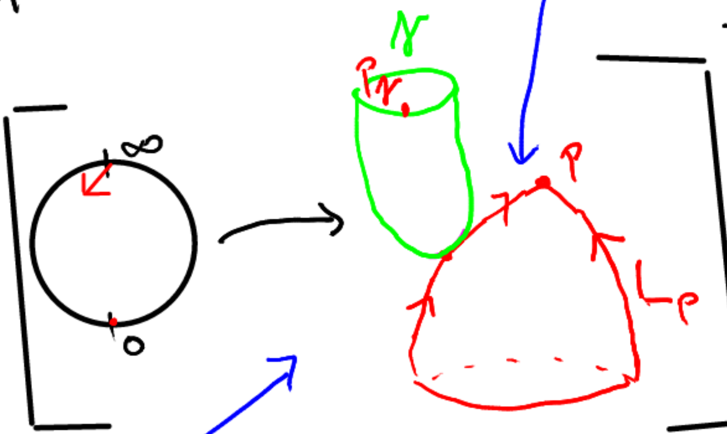
$$d_{SH} = \begin{pmatrix} d_{SH^+} & 0 \\ \delta_{SH} & d_{\mathrm{Morse}} \end{pmatrix}$$

$$d_{SH}^2 = \begin{pmatrix} d_{SH^+}^2 = 0 & 0 \\ \underline{\delta_{SH} d_{SH^+} + \delta_{SH}^2} & d_{Morse}^2 = 0 \end{pmatrix}$$

$$\delta_{SH}\check{\gamma} = \sum_{|\gamma|-|p|=1} \ell_{\gamma p}\, p \,, \quad \delta_{SH}\hat{\gamma} = 0$$

negative gradient flow of $-H$

$$\ell_{\gamma p} = \#_{0\text{-dim}} \Big\{ \Big[\Big] \Big\}$$



no anchors because we are in X

$$=: \underline{\mathcal{M}^X(\gamma;p)}$$

$$\underline{\dim |\gamma|-|p|-1}$$

$$d_{SH}^2 = 0 \implies \underline{SH(X) := H(SH(X), d_{SH})}$$

full symplectic homology

<u>Rem 3.6</u> : $P_{bad} \subset \ker d_{CH}$, $\operatorname{im} d_{CH} \subset \operatorname{span}_{\mathbb{K}} P_{good}$

<u>Exact triangles</u>:

(a)
$$\begin{tikzcd} H^{n-*}(X) \arrow[rr, "0"] & & SH_*(X) \arrow[dl] \\ & SH^+_*(X) \arrow[ul, "\delta_{SH}"] & \end{tikzcd}$$

• tautological triangle.

Also from: $SH = \varinjlim_a HF(H_a)$ (Floer Hofer ...)

From $0 \to \mathrm{Morse}(-H)[-n] \to SH \to SH^+ \to 0$

(b)
$$\begin{tikzcd} CH(X) \arrow[rr, "\delta_{SH^+}", "-2"'] & & CH(X) \arrow[dl, "+1"] \\ & SH^+(X) \arrow[ul, "0"] & \end{tikzcd}$$

as δ_{SH^+} but only good orbits

• Connes (or Gysin)-like triangle $\Rightarrow$ <u>CH equivariant version of SH^+</u>

(z-index filtration: $\mathcal{F}_k SH^+ = \mathrm{span}\{a^\gamma \mid \mu(\gamma) \le k\}$

$E^0_{pq} = \mathcal{F}_p SH^+_{p+q} / \mathcal{F}_{p-1} SH^+_{p+q}$:

q: 0 — 0; 1 — $\widehat{CH}_{p+1}$; 0 — $\check{CH}_p$; $d_0 \downarrow$; p

E^1_{pq} = $\xleftarrow{\hat{d}_{CH}} CH[1]_{p+1}$, $\xleftarrow{\check{d}_{CH}} CH_p$

compute the same hom.

E^2 = $CH_{p-2} \simeq CH_p$, $CH_{p-2} \xleftarrow[d_2]{\delta_{SH^+}} CH_p$

$E^3_{pq} = E^\infty_{pq} = \mathcal{F}_p SH^+_{p+q} / \mathcal{F}_{p-1} SH^+_{p+q}$

E^3 = $E^3_{p-1,1}$ $\mathrm{im}\, d_2$; E^3_{p0} $\ker d_2$; $d_3 = 0$

$SH^+_p = E^\infty_{p0} \oplus E^\infty_{p-1,1}$

$$\begin{tikzcd} \cdots \arrow[r] & SH^+_p \arrow[r] \arrow[dr] & CH_p \arrow[r] & CH_{p-2} \arrow[r] \arrow[dr] & SH^+_{p-1} \arrow[r] & \cdots \\ & & E^3_{p0} \arrow[u] & & E^3_{p-1,1} \arrow[u] & \end{tikzcd}$$

Morphisms induced by cobordisms:

$$F^W_{SH^+} = \begin{pmatrix} F^W_{\check{CH}} & 0 \\ \varphi^W_{SH^+} & F^W_{\widehat{CH}} \end{pmatrix}, \quad F^W_{\check{CH}}, F^W_{\widehat{CH}}$$ defined similarly as $d_{\check{CH}}, d_{\widehat{CH}}$ but using

$\mathcal{M}^W(\gamma;\beta)$ instead

dim $|\gamma|-|\beta|$

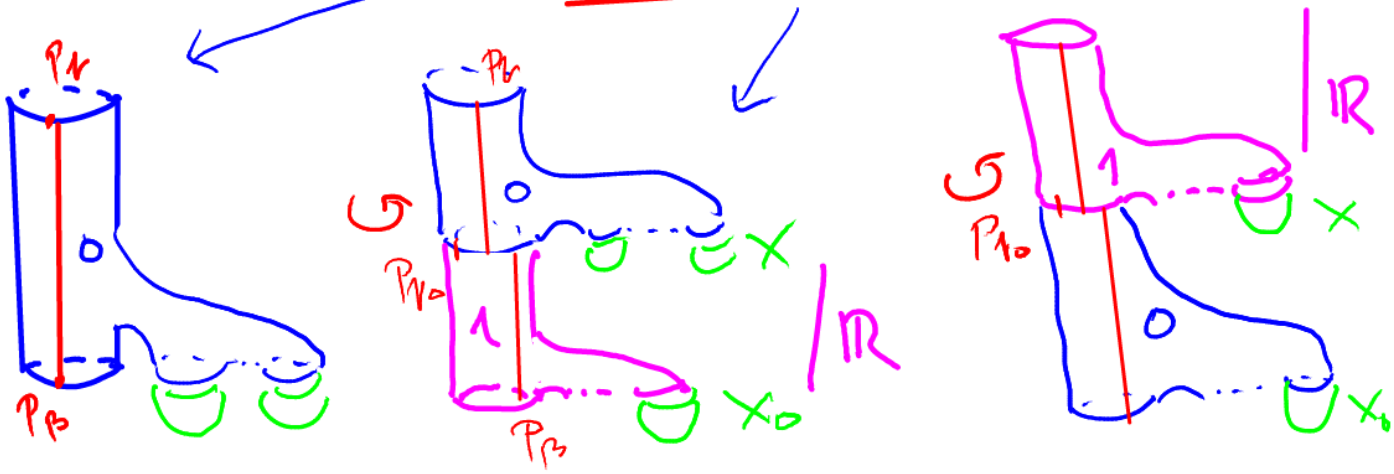
$\#$ 0-dim

anchored in X_0

$\check{\mathcal{M}}^W(\gamma;\beta)$ dim $|\gamma|-|\beta|-1$

$$\varphi^W_{SH^+}\check{\gamma} = \sum_{|\beta|=|\gamma|-1} m_{\gamma\beta}\hat{\beta}$$

$$\check{\mathcal{M}}^W(\gamma;\beta) = \check{\mathcal{M}}'^W \cup \check{\mathcal{M}}''^W \cup \check{\mathcal{M}}'''^W$$



$$d_{SH^+} F^W_{SH^+} = F^W_{SH^+} d_{SH^+}$$

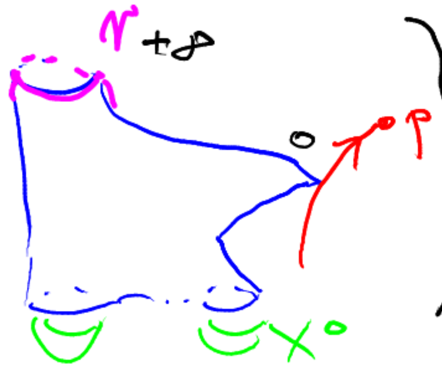
$$\Longrightarrow \quad F^W_{SH^+} : SH^+(X) \longrightarrow SH^+(X_0)$$

$$F^{w}_{SH} = \begin{pmatrix} F^{w}_{SH^+} & 0 \\ \varphi^{w}_{SH} & F^{w}_{Morse} \end{pmatrix}, \quad F^{w}_{Morse} : Morse(-H, X) \longrightarrow Morse(-H, X_0)$$

projection

$$\varphi^{w}_{SH} \check{\gamma} = \sum_{|\gamma| = |p|} \ell_{\gamma p}\, p, \quad \varphi^{w}_{SH} \hat{\gamma} = 0$$

$$\ell_{\gamma p} = \#_{0\text{-dim}} \Big\{ \ \Big\} = M^{w}(\gamma; p)$$



$$d_{SH} F^{w}_{SH} = F^{w}_{SH}\, d_{SH}$$

$$\Longrightarrow \underline{F^{w}_{SH} : SH(X) \longrightarrow SH(X_0)}$$

5. Outlook on BEE

In the situation that X is obtained from X_0 by attaching critical Weinstein handles, enhance $CH(X_0)$, $SH^+(X_0)$, $SH(X_0)$ and F^W_{CH}, $F^W_{SH^+}$, F^W_{SH} by a structure coming from open-closed discs to obtain the following complexes and quasi-isomorphisms:

Legendrian sphere where the handle is attached

$$F^W_{LCH}: CH(X) \longrightarrow CH(X_0) \oplus LH^{cyc}(\Lambda) =: LCH(X_0, \Lambda)$$

$$F^W_{SLH^+}: SH^+(X) \longrightarrow SH^+(X_0) \oplus LH^{Ho^+}(\Lambda) =: SLH^+(X_0, \Lambda)$$

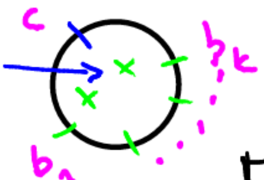
$$F^W_{SLH}: SH(X) \longrightarrow \quad -\|- \quad \oplus \text{Morse}(-H)$$

$$\| \quad SLH(X_0, \Lambda)$$

cyclic and an equivariant version of Hochschild homology of the Chekanov Legendrian DGA $LHA(\Lambda)$

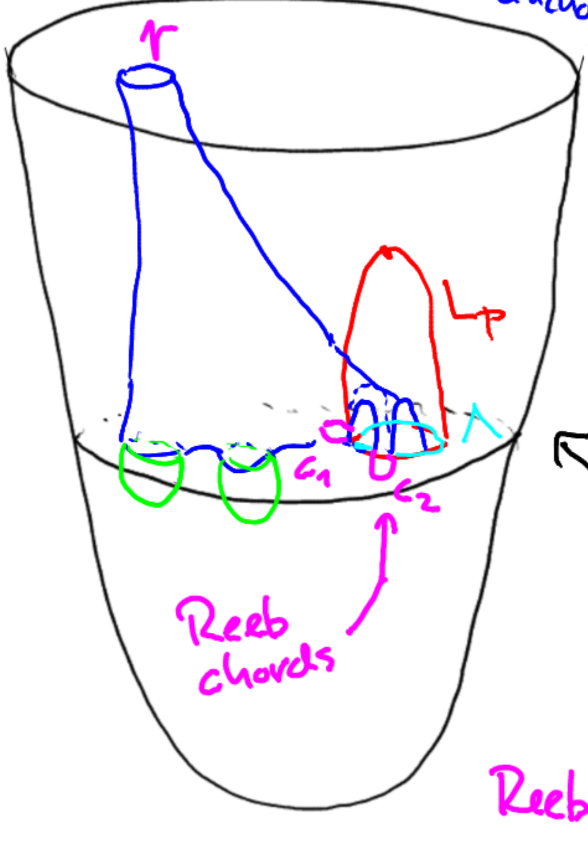
differentials defined by counting open-closed

anchors



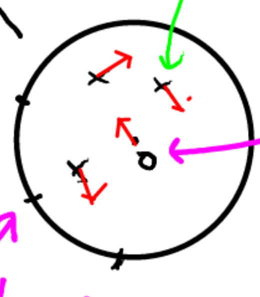
Reeb chords

$\Pi^Y_\Lambda(c; b_1, \ldots, b_k)$



boundary mapped to L_p

anchors



$=: \Pi^W_L(\gamma; c_1, \ldots, c_k)$

(for F^W_{LCH})

γ

disc

Reeb chords