PROBLEM SET 8
To be discussed: 13.12.2017

Instructions

This homework will not be collected or graded, but it is highly advisable to at least think through all of the problems before the next Wednesday lecture after they are distributed, as they will often serve as mental preparation for the material in that lecture. We will discuss the solutions in the Übung beforehand.

Comment: This week's problem set is not as short as it looks, because it has an unusually theoretical focus. The theorem on computation of cellular homology is one of the most important in this course, so its proof is worth learning, and the best way to do that is to work out for yourselves the proofs of similar but slightly different theorems that we haven't seen in lecture. I promise there will be some actual computation practice on the next problem set.

1. Adapt the proof of $H_*^{\mathrm{CW}}(X;G) \cong h_*(X)$ we saw in lecture to prove the relative version of this statement: for any axiomatic homology theory h_* with coefficient group $h_0(\{\mathrm{pt}\}) \cong G$ and any finite-dimensional CW-pair (X, A), i.e. any CW-complex X with a subcomplex $A \subset X$,
$$H_*^{\mathrm{CW}}(X, A; G) \cong h_*(X, A).$$
Explain also how this can be extended to infinite-dimensional CW-pairs when h_* is the singular homology functor $H_*(\,\cdot\,; G)$.
Hint: Consider the groups $h_k(X^n, X^{n-1})$ as before, but with X^n replaced by $X^n \cup A$ for each $n \geqslant 0$.

2. Adapt the proof of $H_*^{\mathrm{CW}}(X;G) \cong h_*(X)$ we saw in lecture to prove the corresponding statement about cohomology: for any axiomatic cohomology theory h^* with coefficient group $h^0(\{\mathrm{pt}\}) \cong G$ and any finite-dimensional CW-complex X,
$$H^*_{\mathrm{CW}}(X;G) \cong h^*(X).$$
Do not worry about the infinite-dimensional case. (For singular cohomology, we will later derive the general case from the universal coefficient theorem.)