Course description and syllabus

General information

Instructors: Prof. Chris Wendl (lectures)
HU Institute for Mathematics (Rudower Chaussee 25), Room 1.301
`wendl@math.hu-berlin.de`
Office hour: Wednesdays 10:30–11:30

Naageswaran Manikandan (problem classes)
HU Institute for Mathematics (Rudower Chaussee 25), Room 1.302
`manikann@hu-berlin.de`

Course webpage: `http://www.mathematik.hu-berlin.de/~wendl/Winter2023/Topologie2/`

Moodle: `https://moodle.hu-berlin.de/course/view.php?id=122954`
enrolment key: `universal`
The moodle will be used mainly for communication: you must join it if you want to receive occasional important announcements about the course via e-mail, and you can also use the moodle forum to discuss or ask questions about homework problems. Essential course materials such as lecture notes will be posted on the course website rather than the moodle.

Lectures: Tuesdays 15:15–16:45 in 1.013 (Rudower Chaussee 25)
Fridays 9:15–10:45 in 1.013 (Rudower Chaussee 25)

Problem classes: Wednesdays 15:15–16:45 in 1-1304 (EWZ, Rudower Chaussee 26)

Language: The course will be taught in English.

Prerequisites: A first course in topology covering the essentials of point-set topology, the fundamental group, Seifert-van Kampen theorem and covering spaces, and some basic knowledge of topological manifolds. Some experience with singular homology as covered e.g. in the last three weeks of the HU's course *Topologie I* in Summer Semester 2023 will also be helpful, though this material will be redone (quickly!) at the beginning of *Topologie II*.

Students who did not take the HU's Topologie I course last semester may find it helpful to skim Lectures 21–25 from the notes for that course (available at `https://www.mathematik.hu-berlin.de/~wendl/Sommer2023/Topologie1/lecturenotes.pdf`) before the start of the semester.

Course description

This is a course in algebraic topology for students with background knowledge as described above under **Prerequisites**. We will develop the singular homology and cohomology functors in depth, with emphasis on the homology of CW-complexes and manifolds, and also their role within the wider context of axiomatic homology theories and their relationship with higher homotopy groups. The tentative program includes as many of the following topics as will fit into one semester:

1. Introduction to categories and functors
2. Main properties of singular homology (homotopy invariance, excision, long exact sequence of the pair)
3. Reduced homology and computation of $H_*(S^n)$
4. Mayer-Vietoris sequence and applications
5. Degree of a map
6. Singular cohomology
7. The Eilenberg-Steenrod axioms for homology and cohomology theories
8. Direct and inverse limits
9. Brief sketch of alternative homology/cohomology theories (Čech and Alexander-Spanier)
10. Axiomatic computation of homology/cohomology for CW-complexes
11. The Lefschetz fixed point theorem
12. Universal coefficient theorem
13. Cross product, cup product and the Künneth formula
14. Topological manifolds, fundamental classes and Poincaré duality
15. Sketch of homological intersection theory for submanifolds
16. Introduction to higher homotopy groups

Literature

Lecture notes for this course will appear in regular updates on the course webpage. They are a continuation of the notes from last semester's *Topologie I* course.[1]

Otherwise, almost everything we will discuss in this course is contained in at least one of the following two books:

- Glen Bredon, *Topology and Geometry*, Springer GTM 1993
  (online access available via the HU library)

- Allen Hatcher, *Algebraic Topology*, Cambridge University Press 2002
  (also freely downloadable from the author's homepage:
  `https://www.math.cornell.edu/~hatcher/AT/ATpage.html`)

Our course will follow Bredon slightly more closely than Hatcher. Here are some other standard algebraic topology books that overlap heavily with each of these:

- James W. Vick, *Homology Theory*, Springer GTM 1994
  (online access available via the HU library)

- R. Stöcker und H. Zieschang, *Algebraische Topologie - Eine Einführung*, Teubner 1994
  (available in the HU library, Freihandbestand)

Finally, the following book is a classic which I cannot recommend as a textbook for learning the material, but its importance as a historical document earns it a place on this list:

- Samuel Eilenberg and Normal Steenrod, *Foundations of Algebraic Topology*, Princton U. Press 1952
  (available in the HU library, Freihandbestand)

[1] The notes will be similar to the notes from my course in 2018–19, which are still available in full at `https://www.mathematik.hu-berlin.de/~wendl/Winter2018/Topologie2/lecturenotes.pdf`.

Exam and problem sets

Grades in the course will be determined by a short **oral exam** soon after the end of the semester (with a resit option shortly before the beginning of the following semester). In the exam, you will need to be able to write down the main definitions in the course, discuss their meaning and significance (with reference to examples where appropriate), and describe the most important applications of the major theorems and the main ideas behind their proofs.

There will be one graded assignment midway through the semester, a so-called **take-home midterm**, which you will have two weeks to work on. Achieving a score of 75% or better on the take-home midterm can boost your final exam grade by one notch, i.e.

- $\geq$ 75% on midterm = (2,0 $\rightarrow$ 1,7 or 1,7 $\rightarrow$ 1,3 etc.)

There will also be ungraded **problem sets** made available every Tuesday and discussed in the problem class on Wednesday of the following week.

Werbung

While the HU *Studienordnung* does not technically contain any course called *Topologie III*, a followup to this course is nonetheless planned for Summer Semester 2024, and will cover some selection from the following topics:

- More on higher homotopy groups (Whitehead, Hurewicz, fibrations and exact sequences)
- Obstruction theory
- Characteristic classes
- Bordism groups