Addendum to A New Fine Structure for Higher Core Models

We here discharge our obligation to show how Steel's proof of iterability for the models N_γ, M_γ which approximate K^c can be modified to give our stronger notion of iterability. This is done in §2. In [S] (Steel's handwritten notes The Core Model Iterability Problem) Steel illustrates his method by dealing with the special case of an iteration of length ω (with some additional simplifying conditions). We redo this case, displaying the necessary modifications. The background condition on extenders which we stated in §11 (and Steel states in [S]) will do for this purpose. We can in fact weaken it. However – as we discuss in §2 – Steel can get by with a still weaker version.

In §1 we bring some definitions which are intended to facilitate the comparison between Mitchell-Steel type mice and ours. Our mice differ in three respects:

(a) Using a somewhat different – but equivalent – fine structure theory, we define a "fine structural ultraproduct" which is applicable without the soundness condition required by Mitchell and Steel.

(b) We index the extenders differently.

(c) We impose a stronger iterability requirement.

In §1 we define a notion of "MS-mouse" which is like ours in respect (a) and (b) and like theirs in respect (c). (We hope that, in the process, we have not done violence to the ideas of Mitchell and Steel. It would of course be equally possible to define "J-mice" which are like theirs in respect (b) and like ours in respect (c).)

The value of this procedure is that we can then ask whether an MS-mouse is also a mouse in our sense.

We don't yet know the answer to this question but we give a partial answer in §3: As usual K^c is constructed in V_θ, where θ is inaccessible. We then – as usual – impose further "bigness requirements" on θ in hopes that K^c will then be "big". In this case our requirements are:

A1 Either no $\xi < \theta$ is Woodin in an inner model or V_θ is closed under #.

A2 Let $Q \in V_\theta$ be a 1-small premouse. Let $\mathcal{I}$ be a normal iteration of Q of length θ. Then $\mathcal{I}$ has a cofinal branch.

A3 θ is a Mahlo cardinal.

(Note A2 holds if $V_\theta^\#$ exists or if θ is not Woodin in an inner model.)

Under these assumptions we show that K^c is universal – i.e. the coiteration of K^c with a 1-small premouse $Q \in V_\theta$ terminates below θ. It follows in particular, that every 1-small $Q \in V_\theta$ which is normally iterable in V_θ is fully iterable in V_θ, since Q coiterates up to a segment of an iterate of K^c. Since MS-mice are normally iterable, we conclude that if $Q \in V_\theta$ is an MS-mouse in V_θ, then Q is a mouse in V_θ. (We can also carry this a bit further, showing fr. inst. that if Q is a countable 1-small countably normally iterable premouse, then Q is a mouse in V_θ. It follows easily that "weak MS-mice" are weak mice in the sense of our §11.)

We refer to our paper "A New Fine Structure ..." as [NFS]. [MS] refers to "Fine Structure and Iteration Trees" by Mitchell and Steel. Section numbers, mention of theorems etc. are to be taken as referring to [NFS] if no other reference is given.

References

[MS] Mitchell, Steel Fine Structure and Iteration Trees

[S] Steel The Core Model Iterability Problem

[NFS] Jensen A New Fine Structure for Higher Core Models