Some Combinatorial Properties of L and V

These notes, presenting results of myself and Kunen, were written at the Rockefeller University in 1969. Much of the material can be found in Frank Drake's book on large cardinals.

Ronald Jensen (Berlin 2002)

Some combinatorial properties of L and V.

The theorems in these notes are due to Jensen and Kunen. In §1 we consider various versions of the combinatorial principles $\diamondsuit$ and $\diamondsuit^+$ for regular $\kappa > \omega$. We show that strong versions of $\diamondsuit^+$ imply correspondingly strong versions of Kurepa's hypothesis. In §2 we define a class of large cardinals which we call <u>ineffable</u>. We show (in ZFC) that $\diamondsuit$ holds for ineffable cardinals, whereas most

versions of $\diamondsuit^+$ fail. In particular, Kurepa's hypothesis fails for inaffable κ. However, if $V = L$ and κ is not inaffable, then all versions of $\diamondsuit^+$ hold. In addition to Kurepa's hypothesis, we consider versions of the combinatorial principle used by Prikry in his paper "On a problem of Gillman + Keisler". Combining our results with Prikry's methods, we get: If $V = L$ and $n < \omega$, then every uniform ultrafilter on $\mathcal{P}(\omega_n)$ is regular. (Prikry had proved this for $n = 1$).

I am grateful to C. C. Chang for suggesting the versions of Kurepa's + Prikry's hypotheses considered here.