Appendix to §8

We again attempt to carry out our proof for arbitrary mice, using Neeman - Steel in place of Dodd-Jensen. If M is a countable mouse and $\vec{\eta} = \langle \eta_i \mid i < \omega \rangle$ an enumeration of M, we say that an iteration strategy is $\vec{\eta}$ - minimal if it has the properties given by the Neeman - Steel lemma. We say that S is minimal if it is $\vec{\eta}$ - minimal for some $\vec{\eta}$. Now let $\sigma : W \to M$ witness the goodness of $\langle M, W, \lambda \rangle$. We say that a coiteration $\langle \mathcal{I}^W, \mathcal{I}^M \rangle$ of $\langle M, W, \lambda \rangle$ against M is minimal iff the copy $\sigma(\mathcal{I}^W)$ exists and $\sigma(\mathcal{I}^W)$, $\mathcal{I}^M$ employ the same minimal strategy. Lemma 1 then holds in the obvious reformulation:

Lemma 1′ Let M be a countable mouse. Let $\sigma : W \to M$ witness the goodness of $\langle M, W, \alpha \rangle$. Let $\langle \mathcal{J}^W, \mathcal{J}^Q \rangle$ be a minimal coiteration of $\langle M, W, \alpha \rangle$ against M. Let $\theta = |\mathcal{J}^Q|$. Then:

(a) $\theta \geq 0$ in $\mathcal{J}^W$.

(b) W_θ is a simple iterate of W in $\mathcal{J}^W$ and a segment of Q_θ.

(c) W is a mouse.

. . . . .

The proof of Lemma 2 goes through as before: By Löwenheim–Skolem we take M as countable. The double rooted coiteration in the proof is then taken as minimal. Lemmas 3.1, 3.2 hold as before. In the proof of Lemma 4 we again (w.l.o.g.) take M as countable and apply a minimal double rooted coiteration. (We induct on the relation:

NRN' iff N is a countable nonsimple iterate of N'.) At two places, however, we used a property of mice which was derived from basicness:

(*) Let $N = \langle J_\nu^E, F \rangle$ be a mouse with $F \neq \emptyset$. Then $\omega\rho_N^1 < \nu =$ the largest cardinal in N.

This was used in Case 2.2 (to prove that $\nu_0 > On \cap \bar{M}$) and in (1) of Case 2.3. Thus, to carry out the proof we must assume:

(**) Every segment of M satisfies (*).

There is no doubt that we can go very far using only mice which satisfy (**). The existence of a counterexample to (**) has, in fact, not been shown, even using very large cardinals. However, even if we drop (**), we can still get a weaker version of Lemma 4. (c) must then be replaced by:

(c′) $\bar{M}$ is a segment of H, where $\pi : M\|\eta \longrightarrow_{E^M_\mu} H$ for some $\eta \in M$, $\mu \leq \eta$.

The weakening is necessary only if $\nu_0 > On \cap \bar{M}$ fails in Case 2.2 or (1) of Case 2.3 fails. If that happens, we still get (c′) with $\eta = \mu = \nu$ and $\bar{M}$ is a <u>proper</u> segment of H. (We omit the proof.) But then $\omega\rho^{\omega}_{\bar{M}} \geq \nu$, since ν is a cardinal in H. Thus the full conclusion of Lemma 4 will still hold if $\omega\rho^{\omega}_{\bar{M}} < \nu$.

The proof of Lemma 4.1 goes through as before with the obvious modifications. However, we used §6 Cor 7.1, which was a strong consequence of basicness. Hence Lemma 4.1 is proven only for mice M all of whose segments satisfy §6 Cor 7.1. The proof of Cor 4.2 goes through for arbitrary mice. Cor 4.3 goes thru for M satisfying (*) (4.2 and 4.3 show how difficult it will be to find mice not satisfying (*).) Lemma 5 goes through in full generality.

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