1 Assumptions in causal inference

a) We investigate the effect of a specific motivational training on the performance of students in elementary school. We recruit several school classes and in each class, we randomly allocate one half of the students to the motivational training, while the other half serves as the control group.

   (i) It is likely that the students in the motivation group share what they learned in the motivational training with the other students. Which assumption is violated?

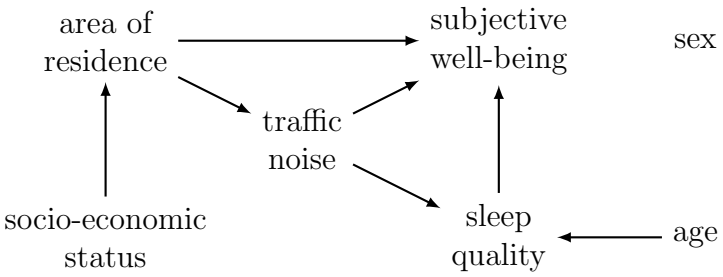
   (ii) Name sources of (unobserved) confounding in this study, if any.

b) We are interested in the effect of antihypertensives (drugs that lower the blood pressure) on the incidence of chronic heart disease (CHD). Our data base contains information on prescribed antihypertensives, CHD and further health-related as well as demographic information for a large number of patients over time.

   (i) There are several different classes of antihypertensives that lower the blood pressure by different means. Some of them are never prescribed to patients with specific comorbidities (e.g. diuretics are contra-indicated in patients suffering from chronic obstructive pulmonary disease). Which assumption(s) may be violated? How could the research question be modified?

   (ii) Name sources of (unobserved) confounding in this study, if any.

2 Causal DAGs and regression



Suppose the above DAG is causal.

a) Which statements are correct?

   (i) In a regression of 'subjective well-being' on 'age' and 'sex', we expect both coefficients to be zero.

   (ii) The coefficients in a regression of 'sleep quality' on 'traffic noise', 'subjective well-being' and 'age' have a causal interpretation.

   (iii) The coefficients in a regression of 'subjective well-being' on 'traffic noise' and 'sleep quality' have a causal interpretation.

   (iv) If we regress 'sleep quality' on 'age' and obtain a coefficient of about zero, we know that the DAG is wrong.

b) Suppose we are interested in the total causal effect of 'sleep quality' on 'subjective well-being.' For what confounders do we need to adjust? How could the desired effect be obtained?

3 (Augmented) IPTW

a) Consider an exposure X (binary), a sufficient set of covariates C (discrete), and an outcome Y; assume $\pi(c) = P(X = 1|C = c)$ is known.
Show that $E(XY/\pi(C)) = E(Y|\text{do}(X = 1))$ or $E(XY/\pi(C)) = E(Y(1))$. State clearly on what assumptions you rely. (Trick: use iterated conditional expectation.)

b) In the same situation as above:

(i) show that for

$$\hat{\mu}_1 = m(C) + \frac{X}{\pi(C)}(Y - m(C))$$

we have: $E(\hat{\mu}_1) = E(Y|\text{do}(X = 1))$ if either $m(C) = E(Y \mid X = 1, C)$ or if $\pi(c) = P(X = 1|C = c)$.

(ii) What does the corresponding $\hat{\mu}_0$ have to look like?

4 Adjustment using standardisation and IPTW by hand

The following examples are from the 'Causal Inference' book by Miguel Hernán and Jamie Robins.

	$Y(0)$	$Y(1)$
Rheia	0	1
Kronos	1	0
Demeter	0	0
Hades	0	0
Hestia	0	0
Poseidon	1	0
Hera	0	0
Zeus	0	1
Artemis	1	1
Apollo	1	0
Leto	0	1
Ares	1	1
Athena	1	1
Hephaestus	0	1
Aphrodite	0	1
Cyclope	0	1
Persephone	1	1
Hermes	1	0
Hebe	1	0
Dionysus	1	0
Σ	10	10

	X	C	Y
Rheia	0	0	0
Kronos	0	0	1
Demeter	0	0	0
Hades	0	0	0
Hestia	1	0	0
Poseidon	1	0	0
Hera	1	0	0
Zeus	1	0	1
Artemis	0	1	1
Apollo	0	1	1
Leto	0	1	0
Ares	1	1	1
Athena	1	1	1
Hephaestus	1	1	1
Aphrodite	1	1	1
Cyclope	1	1	1
Persephone	1	1	1
Hermes	1	1	0
Hebe	1	1	0
Dionysus	1	1	0
Σ	13	12	10

a) The **left**-hand table contains data for Greek gods waiting for heart transplants. The variable X describes the treatment ($X = 0$ no transplant received, $X = 1$ transplant received). The outcome of interest is survival five years after treatment ($Y = 0$ alive, $Y = 1$ dead). By divine revelation we know the potential outcomes for each of the gods.

   (i) Is there an individual causal effect for Zeus?

   (ii) What is the average causal effect? (causal risk difference)

b) The **right**-hand table contains related data from an observational study. Shown are the actual treament X, the actual outcome Y and a risk factor C. Assume that within strata of C, the treatment X can be considered like randomised. Fill in the missing numbers in the following (the gaps are labelled for easier reference).

   (i) The crude (unadjusted) difference is
   $\Delta = \hat{P}(Y = 1 \mid X = 1) - \hat{P}(Y = 1 \mid X = 0) =$_____(1).

   (Compare this to the average causal effect from a)(ii) – association is not causation!)

(ii) Estimate the average causal effect via standardisation (see slide 46).
Subgroup with $C = 0$:
The estimated risk of dying when treated is
$\hat{P}(Y = 1 \mid X = 1, C = 0) =$_____(2).
The estimated risk of dying when untreated is
$\hat{P}(Y = 1 \mid X = 0, C = 0) =$_____(3).

Subgroup with $C = 1$:
The estimated risk of dying when treated is
$\hat{P}(Y = 1 \mid X = 1, C = 1) =$_____(4).
The estimated risk of dying when untreated is
$\hat{P}(Y = 1 \mid X = 0, C = 1) =$_____(5).

Standardisation:
The estimated standardised risk of dying for the whole population when treated is
$\hat{P}(Y(1) = 1)$
$= \hat{P}(Y = 1 \mid X = 1, C = 0) \cdot \hat{P}(C = 0) + \hat{P}(Y = 1 \mid X = 1, C = 1) \cdot \hat{P}(C = 1)$
=_____(6).
The estimated standardised risk of dying for the whole population when untreated is
$\hat{P}(Y(0) = 1)$
$= \hat{P}(Y = 1 \mid X = 0, C = 0) \cdot \hat{P}(C = 0) + \hat{P}(Y = 1 \mid X = 0, C = 1) \cdot \hat{P}(C = 1)$
=_____(7).
The estimated average causal effect is
$\hat{ACE} = \hat{P}(Y(1) = 1) - \hat{P}(Y(0) = 1) =$_____(8).
(Compare this to the average causal effect from a)(ii).)

(iii) Estimate the average causal effect via inverse probability of treatment weighting (see slide 57ff.).
The estimated probability of treatment given $C = 0$ is
$\hat{P}(X = 1 \mid C = 0) =$_____(9).
The estimated probability for having $C = 0$, being treated and dying is
$\hat{P}(C = 0, X = 1, Y = 1) =$_____(10).
Weighting with the inverse probability of treatment given $C = 0$ yields
$\frac{\hat{P}(C=0,X=1,Y=1)}{\hat{P}(X=1|C=0)} =$_____(11).
The estimated probability for having $C = 0$, not being treated and dying is
$\hat{P}(C = 0, X = 0, Y = 1) =$_____(12).
Weighting with the inverse probability of no treatment given $C = 0$ yields
$\frac{\hat{P}(C=0,X=0,Y=1)}{\hat{P}(X=0|C=0)} =$_____(13).

The estimated probability of treatment given $C = 1$ is
$\hat{P}(X = 1 \mid C = 1) =$_____(14).
The estimated probability for having $C = 1$, being treated and dying is
$\hat{P}(C = 1, X = 1, Y = 1) =$_____(15).
Weighting with the inverse probability of treatment given $C = 0$ yields
$\frac{\hat{P}(C=1,X=1,Y=1)}{\hat{P}(X=1|C=1)} =$_____(16).

The estimated probability for having $C = 1$, not being treated and dying is $\hat{P}(C = 1, X = 0, Y = 1) =$_____(17).
Weighting with the inverse probability of no treatment given $C = 1$ yields $\frac{\hat{P}(C=1,X=0,Y=1)}{\hat{P}(X=0|C=1)} =$_____(18).

The estimated weighted risk of dying for the whole population when treated is $\hat{P}(Y(1) = 1) = \frac{\hat{P}(C=0,X=1,Y=1)}{\hat{P}(X=1|C=0)} + \frac{\hat{P}(C=1,X=1,Y=1)}{\hat{P}(X=1|C=1)} =$_____(19).
The estimated weighted risk of dying for the whole population when not treated is
$\hat{P}(Y(0) = 1) = \frac{\hat{P}(C=0,X=0,Y=1)}{\hat{P}(X=0|C=0)} + \frac{\hat{P}(C=1,X=0,Y=1)}{\hat{P}(X=0|C=1)} =$_____(20).
The estimated average causal effect is
$\hat{ACE} = \hat{P}(Y(1) = 1) - \hat{P}(Y(0) = 1) =$_____(21).
(Compare this to the average causal effect from a)(ii).)

5 Instrumental Variables

Consider the standard IV set-up with instrument G, exposure X, outcome Y, unobserved confounder U, and assume that the IV conditions are satisfied.

a) Assume all observable variables G, X, Y are binary.

   (i) Use a SWIG to show that $Y(x) \perp\!\!\!\perp G$.

   (ii) Show that $E(Y(1) - Y(0)|X = 1, G = g) = \psi$ is equivalent to

$$E(Y|X = x, G = g) - E(Y(0)|X = x, G = g) = \psi x.$$

   (iii) Use (i) and (ii) to show that

$$\psi = \frac{E(Y|G = 1) - E(Y|G = 0)}{E(X|G = 1) - E(X|G = 0)}.$$

   Trick: take expectation over X given $G = g$.

b) Now, for continuous Y, assuming

$$E(Y|X = x, U = u) = \mu_Y + \beta x + h(u),$$

   show that

$$\beta = \frac{Cov(Y, G)}{Cov(X, G)}.$$

   State clearly what IV assumptions you use.
   Trick: define $\tilde{G} = G - E(G)$ and work out $E(Y\tilde{G})$.

c) Typical data, where IVs might be useful, are obtained from case-control studies: this means that 50% of the observations were sampled from known 'cases' $Y = 1$ and the other 50% from known 'controls' $Y = 0$.

   (i) Draw a DAG that includes a sampling indicator S to represent this situation.

   (ii) Give arguments for or against the validity of IV-based inference regarding (I) testing the null-hypothesis of no $X \to Y$ edge; (II) estimating the causal effect of X on Y using G with a standard IV-method.