WINTERSEMESTER 2015/16 - NICHTLINEARE PARTIELLE DIFFERENTIALGLEICHUNGEN

Homework #6 due 11/27/2015

Problem 1. a.) Show that the Dirichlet problem for the Laplace operator satisfies the Lopatinskii condition. Prove $\dim\ker T = 0$. (Here T is the operator defined in (2.6.1) in the Lecture Notes.)

b.) Show that the Neumann problem for the Laplace operator in $\Omega \subset \mathbb{R}^d$ satisfies the Lopatinskii condition. Prove $\dim\ker T = 1$.

Problem 2. Consider the 4×4 first-order differential operator in $\mathbb{R}^3_+$

$$P(\partial)u = \begin{bmatrix} \nabla \times v + \nabla w \\ -\nabla \cdot v \end{bmatrix} .$$

Here $u = \begin{bmatrix} v \\ w \end{bmatrix}$ is a vector-valued function with four components, v is a vector-valued function with three components, and the function w is scalar-valued.

a.) Show that the boundary condition $B_1 u = n \times v$ satisfies the Lopatinskii condition. Here and henceforth $n = -e_3$ is the exterior unit normal vector of $\mathbb{R}^3_+$ along $\partial\mathbb{R}^3_+$.

b.) Show that the boundary condition $B_2 u = (n \cdot v, w)$ satisfies the Lopatinskii condition.

c.) Do these two boundary condition satisfy the Lopatinskii condition with respect to the operator

$$P_\alpha(x, \partial)u = \begin{bmatrix} \nabla \times v + \alpha(x)\nabla w \\ -\nabla \cdot (\alpha(x)v) \end{bmatrix}$$

already considered in Homework #3? Suppose that α is a 3×3 Hermitian matrix.

Problem 3. Can you find a boundary condition for the system in Problem 2 that does not satisfy the Lopatinskii condition ?