WINTERSEMESTER 2015/16 - NICHTLINEARE PARTIELLE DIFFERENTIALGLEICHUNGEN

Homework #9 due 12/19/2015

Problem 1. Consider the homogeneous isotropic elastic wave equations with constant coefficients, that is

$$\rho \frac{\partial^2 u}{\partial t^2} - \nabla \cdot [\mu(\nabla u + \nabla u^T)] - \nabla[\lambda \nabla \cdot u] = 0 \, .$$

Here ρ is the density, μ and λ are the Lamé parameters, and $u : [0, \infty) \times \mathbb{R}^3 \to \mathbb{R}^3$ denotes the displacement. All three coefficients are assumed to be positive constant.

a.) Find solutions of the form $u(t, x) = ae^{\omega t - k \cdot x}$ where ω is a positive constant and $a, k \in \mathbb{R}^3$. These solutions are called plane waves.

b.) Characterize your plane wave solutions as longitudinal ($k \parallel a$) or transversal ($k \perp a$).

Problem 2. Suppose that P is symmetric hyperbolic and that $A^0 = I_N$. Show that $u \in L_2(Q)$ and $Pu \in L_2(Q)$ implies that $u(t, \cdot) \in H^{-1/2}(\mathbb{R}^d)$ for all $t \in [0, T]$. Recall that $Q = (0, T) \times \mathbb{R}^d$.

Problem 3. Prove the following simplified version of Lemma 3.2.1: A function $u \in L_\infty(Q)$ satisfies $u \in W^1_\infty(\mathbb{R}^d)$ if and only if u is Lipschitz, i.e., there exists a constant $L > 0$ such that $|u(x) - u(y)| \leq L|x - y|$ for all $x, y \in \mathbb{R}^d$. (Hint: Use the regularization of functions as decribed in the lecture note before Lemma 3.3.4.)