SOMMERSEMESTER 2016 - HÖHERE ANALYSIS II
LINEARE PARTIELLE DIFFERENTIALGLEICHUNGEN

Homework #11 due 07/05

Problem 1. Suppose that $K : H \to H$ is a compact linear operator on a Hilbert space H. Prove that
a.) $N(I-K) = \{x \in H \ : \ Kx = x\}$ is finite-dimensional.
b.) $R(I-K) = \{x - Kx \ : \ x \in H\}$ is closed. Hint: Show at first that there exists a constant $\gamma > 0$ such that

$$\|u - Ku\| \geq \gamma \|u\| \quad \text{for all } u \in N(I-K)^{\perp} .$$

Problem 2. For $f \in L_2(\mathbb{R}^d)$ use the Fourier transform in space to derive a solution formula for the initial value problem to the heat equation

$$\begin{aligned} u_t - \Delta u &= 0 \quad \text{in } (0,\infty) \times \mathbb{R}^d , \\ u(0,\cdot) &= f \quad \text{in } \mathbb{R}^d . \end{aligned}$$

Problem 3. Suppose that $u \in H^s(\mathbb{R}^d)$ for some $s > 1/2$. Show that the mapping $T : C_0^\infty(\mathbb{R}^d) \to C_0^\infty(\mathbb{R}^{d-1})$ given by $Tu(x', x_d) = u(x', 0)$ extends to a continuous linear operator from $H^s(\mathbb{R}^d)$ into $H^{s-1/2}(\mathbb{R}^{d-1})$.