SOMMERSEMESTER 2016 - HÖHERE ANALYSIS II
LINEARE PARTIELLE DIFFERENTIALGLEICHUNGEN

Homework #12 due 07/12

Problem 1. Suppose that $K : H \to H$ is a compact linear operator on a Hilbert space H and K^* is the adjoint. Prove the following statements.
a.) $N(I - K^*)^\perp = R(I - K)$. Hint: Make use of Problem 1 from the previous week.
b.) $R(I - K) = H$ if and only if $N(I - K) = \{0\}$.

Problem 2. For $f \in L_2(\mathbb{R}^d)$ use the Fourier transform in space to derive a solution formula for the initial value problem to the Schrödinger equation

$$\begin{aligned} iu_t - \Delta_x u &= 0 \quad \text{in } (0,\infty) \times \mathbb{R}^d \,, \\ u(0,\cdot) &= f \quad \text{in } \mathbb{R}^d \,. \end{aligned}$$

Here $i = \sqrt{-1}$.

Problem 3. Find a fundamental solution to the Schrödinger operator, that is a distribution $\Phi(t,x) \in \mathscr{D}'(\mathbb{R}^{d+1})$ that satisfies

$$i\partial_t \Phi = \Delta_x \Phi \text{ for } t \neq 0, x \in \mathbb{R}^d \quad \text{and} \quad \Phi(0,x) = \delta_0 \text{ for } x \in \mathbb{R}^d \,.$$

Hint: Use Problem 2 and Lemma 7.2, which also explains how to understand the condition $\Phi(0,x) = \delta_0$.