SOMMERSEMESTER 2016 - HÖHERE ANALYSIS II
LINEARE PARTIELLE DIFFERENTIALGLEICHUNGEN

Homework #7 due 06/07

Problem 1. Prove that $C_0^\infty(\mathbb{R}^d)$ is dense in $H^k(\mathbb{R}^d)$ for all positive integers k and d.

Problem 2. Give a counterexample for the Poincaré inequality (Theorem 5.1) in $\mathring{H}^1(\mathbb{R}) = H^1(\mathbb{R})$. Construct a sequence $u_n \in C^\infty(\mathbb{R})$ of functions such that $\int_{\mathbb{R}} |u_n'(x)|^2\, dx \to 0$ while $\int_{\mathbb{R}} |u_n(x)|^2\, dx = 1$ for $n \to \infty$.

Problem 3. Rellich selection theorem. *Suppose that Ω is a bounded, connected, and open set of class C^1. Then every bounded sequence in $H^1(\Omega)$ has a strongly converging subsequence in $L_2(\Omega)$.*
Use this theorem to prove the Poincaré inequality $\|u\|_{L_2(\Omega)} \leq C\|\nabla u\|_{L_2(\Omega)}$ for
a.) all $u \in \mathring{H}^1(\Omega)$.
b.) all $u \in H = \{u \in H^1(\Omega)\ :\ \int_\Omega u\, dx = 0\}$.
Hint: Argue by contradiction.